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Transfer of Learning

Progressive Perspectives for Mathematics Education and Related Fields

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Editors

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Chapter 2
An Embodied Theory of Transfer of Mathematical Learning

Mitchell J. Nathan and Martha W. Alibali

In the brief photo transcript shown in Table 2.1, below, taken from a high school engineering lesson, we encounter a critical educational challenge: In the rich sensory stream of spoken words and metaphors, written symbols, diagrams and sketches, gestures, simulations, and actions on objects, all of which occur in multiple venues such as the classroom and machine shop, how do learners perceive and construct for themselves a connected meaning of a concept such as *theta*, the angle of ascent of a projectile? The answer, we argue, depends on a theory of transfer that is *embodied*: The concept is depicted and comprehended in terms of actions, gestures, spatial metaphors, and other body-based resources; embedded in various specific physical and social settings; extended across multiple modalities, material resources and participants; and enacted through the actual or simulated interplay of perception and action among students and their teachers.

Project-based learning (PBL) environments, such as those common to problem-based and other science, technology, engineering, mathematics (STEM) education settings, offer a rich stream of activities and experiences that are intended to ground students' understanding of important mathematical ideas and to motivate the relevance of these ideas across a range of content and contexts. In so doing, success in PBL settings requires learners to construct a concept—such as THETA—and follow it across a multitude of modal forms and contexts while recognizing it as *invariant*. Understanding what is required of students to establish, perceive, maintain, and express such *invariant relations* across such environmental and perceptual variability motivates an embodied theory of transfer of mathematical knowledge.

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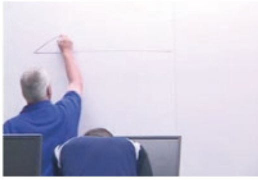
Table 2.1 Photo Transcript 1: Day 1

Teacher (to class): What happens when the, when we **project something through the air [1]**, is we end up with **something like [2] this** depending upon the angle here, which is *theta*. And, this is our range. And basically, what we have, is, we're working with vectors here. So, we end up with, some vectors that look like this and we call this, vector V_x and V … V_y. And we can say that, V_y we're gonna start with V_y here. This distance, this, right here. So, we're gonna start with, V_y, equals V_o, sine, of *theta*. Student: Mr. [Name], what's V_o?	 [1]  [2]

Note. Bold text and indices align with images

In this chapter, we first argue that the processes involved in establishing and maintaining cohesion of invariant relations during PBL are not readily described by classic accounts of transfer. We hypothesize instead that the processes involved in transfer of mathematical ideas throughout complex learning settings are necessarily embodied, and we consider the assumptions that form the basis of an embodied account of transfer as mapping of modes of perceiving and acting to achieve cohesion across contexts. We then use this embodied framework to illustrate successful and unsuccessful transfer in PBL settings. From this, we propose that transfer processes are necessarily embodied and socially mediated, in that they are grounded in the actions on and perceptions of the material world in which they are embedded and they are extended across multiple actors, typically learners and their teachers. These elements come together in an embodied theory of transfer. In the final section, we discuss the implications of embodied transfer for educational practice and identify important future areas for research.

2.1 Limitations of Classic Approaches of Transfer

Transfer can be defined as the application and extension of learned mathematical ideas beyond the context in which they were originally learned. Transfer has a long history in educational psychology (Bransford & Schwartz, 1999; Woodworth & Thorndike, 1901). Indeed, the enterprise of a liberal arts education is predicated on the notion of transfer and on the idea that learning general topics and principles will provide guidance for addressing the social and scientific issues facing the next generation of leaders, scholars, artisans, and others.

At the heart of classical models of transfer lies the notion of *common elements*, wherein the transfer of skilled performance is modeled as reapplication of previously learned actions that follow from an expert's assessment of the degree of overlap of environmental conditions that may be readily observed (near transfer) or that are only apparent at a deeper, structural level (far transfer; Singley & Anderson, 1989; Taatgen, 2013). From this theoretical perspective, abstracted, rule-like condition-action processes are antecedent to successful transfer. In this sense, perceptual richness is antithetical to transfer because it works against the formation of abstractions and their reapplication (e.g., Kaminski, Sloutsky, & Heckler, 2013).

Classical accounts of transfer fall short at explaining PBL and the learning that occurs in complex settings in several respects. First, the common elements that are the signature of classical accounts of transfer are often identified by experts, rather than generated from the learner's perspective (Lobato, 2003, 2006). Thus, it is not clear whether learners are aware of them and actually transferring on the basis of those common elements. Second, classical accounts are founded on analyses of simple stimuli, for which identifying common elements is relatively straightforward. This is not the case in many PBL settings, in which a single curriculum unit can extend over long periods of time in multiple spaces; can include many participants; and can engage a variety of objects, technological resources, and notational systems (Kozma, 2003). Third, classical accounts foreground learners' transfer processes while marginalizing (or neglecting entirely) the pedagogical processes enacted by teachers that establish the contexts in which transfer takes place and that support processes of transfer.

A primary issue for students in PBL is having a cohesive experience so that the various elements of the learning environment are experienced as connected and meaningful (Nathan, Wolfgram, Srisurichan, Walkington, & Alibali, 2017). *Cohesion* is the quality of unity or relatedness of ideas and experiences. It is commonly operationalized in terms of the degree to which ideas in a complex text are interconnected, even as one moves across clauses and sentences (McNamara, Graesser, Cai, & Kulikowich, 2011). As used here, producing cohesion refers to forming and maintaining connections among the many disparate elements of the learning environment that might otherwise serve as obstacles to transfer. For engagement and learning to take place in PBL settings, cohesion of invariant relations is paramount because ideas are presented in a variety of forms and settings. However, this process has been neglected in classical accounts of transfer.

2.2 Transfer as Embodied: Underlying Assumptions

Numerous challenges and alternatives to the classical theory of transfer have been raised, addressing the reductionist basis of transfer and the insensitivity of the classical theory to situated context (e.g., Detterman & Sternberg, 1993) and culture (e.g., Scribner & Cole, 1981). For example, the situative perspective (Greeno, Smith, & Moore, 1993) privileges participation across contexts over the reapplication of knowledge in assessing transfer. The actor-oriented transfer perspective (Lobato, 2003, 2008) considers generalized behaviors based on the agent's perspective of what is similar across familiar and novel settings. An account based on learners' "episodic feelings" integrates cognition, emotion, and bodily experiences in explaining patterns of transfer (Nemirovsky, 2011). These alternative frameworks share a view of the learner as an embedded, engaged, embodied actor and allow for the world to "seep in" to the cognitive realm that was previously theorized as isolated from the material realm.

With advancements in theories of embodied cognition, there is now a sufficient conceptual and empirical foundation for articulating an embodied theory of transfer of mathematical ideas. Indeed, we argue that an embodied perspective is necessary to account for learners' transfer in PBL settings and other complex learning environments, and it can also help explain instances of unsuccessful transfer. A related idea was presented by Goldstone, Landy, and Son (2008), who theorized that learning grounded in perception and interaction supports generation of transferable knowledge. They demonstrate successful transfer on tasks such as solving symbolic algebra equations and understanding the cross-domain application of deep principles of complex systems performance. Based on their analyses of these examples, they propose that perceptual knowledge transfers

> to new scenarios and transports across domains, most often proceeding not through acquiring and applying symbolic formalisms but rather through modifying automatically perceived similarities between scenarios by training one's perceptual interpretations. (p. 329)

This account of the role of perception and interaction in transfer is promising for a broad account of transfer to complex, collaborative, multimodal learning contexts.

There are several assumptions at the core of our embodied theory of transfer. The first assumption is that the cognitive system is a *predictive architecture*. Rather than passively waiting for input to act, humans are continually anticipating the next events in the stream of sensory input and are already poised to respond. In this sense, transfer is the default mode—no two environmental stimuli are identical, and, regardless, body states are never fixed in time. Whether transfer is deemed successful is often a function of experts' expectations for what should be transferred, rather than whether any form of transfer took place for the learner.

Second, there is reciprocity between cognitive states and actions, such that actions (arm movements performed by a student, for example) can drive the system into related cognitive states through the process of *action-cognition transduction* (Nathan, 2017; Nathan & Walkington, 2017). Transduction provides an account for how systems can operate in "forward" and "reverse" directions, a common property of many physical and biological systems. For example, with cognition driving action in the "forward" direction, a student may spontaneously extend her arms in a mathematically relevant manner to assist her in reasoning about a property of triangles. Students can also be prompted to extend their arms in either a mathematically relevant or irrelevant manner by having them touch locations on an interactive whiteboard. Nathan and colleagues (Nathan et al., 2014) investigated the hypothesis that mathematically relevant movements would drive the cognition-action system in the "reverse" direction and activate the appropriate conceptual reasoning for the task, but mathematically irrelevant movements would not assist the student. In support of this hypothesis, they found the mathematically relevant movements improved mathematical proof production, even though participants reported making no connection between the mathematics and the directed movements, and mathematically irrelevant movements did not.

Transduction recognizes that actions can drive the system to certain cognitive states using many of the same pathways that enable cognitive processes to elicit actions. Transduction plays an integral role in explaining successful and unsuccessful transfer. It explains, for example, how the execution of previous modes of perceiving and acting, activated by familiar contextual cues or expectations of the predictive architecture, can activate inappropriate concepts, leading to unsuccessful transfer to new task demands. Our focus on transduction reflects the empirically supported view that the coupling between cognition and action involves rich, multidirectional pathways (e.g., Abrahamson & Trninic, 2015; Nathan et al., 2014; Thomas, 2013)—richer than those that are typically described in classical information processing theory, which generally acknowledges only a unidirectional pathway via which cognition drives actions.

Third, people do not come to know the world as a verbatim sensorial record of an objective external world; instead, people are driven to make sense of their experiences, and meaning is constructed through the continuous interplay of social, cognitive, motoric, and perceptual processes of a highly dynamic, self-regulating organism, in what is often referred to as the *perception-action loop* (cf. Neisser's (1976) "perceptual cycle" as being central to everyday cognition). People construct mathematical meanings by coordinating situated perceptual and motor behaviors with the behaviors of mathematical objects (Abrahamson & Sánchez-García, 2016). Thus, the world we can know depends in part on the ways in which we can interact with it, physically and perceptually (Varela, Thompson, & Rosch, 1991). Meaning making also depends on establishing and maintaining common ground among interlocutors (e.g., H. H. Clark & Schaefer, 1989; Nathan, Alibali, & Church, 2017). Embodied processes are crucial for efforts to manage common ground in pedagogical contexts, where teachers regularly strive to foster common ground by using indexical speech and linking gestures (e.g., Alibali et al., 2014; Alibali, Nathan, Boncoddo, & Pier, 2019).

Fourth, mathematical ideas are *embodied* and *tangible* (Hall & Nemirovsky, 2012), and they can be expressed in metaphorical speech (Lakoff & Núñez, 2000), gestures and simulated actions (Hostetter & Alibali, 2008, 2019), diagrams and inscriptions (de Freitas & Sinclair, 2014), and physical objects (Martin & Schwartz, 2005). Importantly, mathematical ideas in different modalities may be linked via speech, gestures, and action (Goodwin, 2013), creating a rich multimodal experience that is a signature of PBL and that serves to ground the meanings of the referents.

Fifth, cognition is *extended* beyond the individual actor's brain such that task-relevant knowledge is grounded and distributed across actors, objects, and space (A. Clark & Chalmers, 1998). One example is cognitive offloading, wherein actors "use the world as its own model" (Brooks, 1991, p. 139) rather than depend on symbolic representations of the world and symbol-manipulation operations on those representations, which are the hallmark of traditional transfer (e.g., Lave, Murtaugh, & de la Rocha, 1984).

Sixth, because transfer is *embedded* in the situations in which activity unfolds, teachers and students are each engaged in transfer, and they serve as actors in exchanges that are situated in particular learning contexts. In many cases, teachers and curriculum developers have thoughtfully designed specific contextual supports for transfer; in other cases, teachers generate such supports "on the spot."

Finally, conceptual development naturally follows a process of *progressive formalization* (Romberg, 2001), which can be instantiated in the pedagogical practice of *concreteness fading* (Fyfe, McNeil, Son, & Goldstone, 2014). Concreteness fading is a developmentally informed approach to instruction that recognizes the importance of initial physical interactions (enactive processes) for early sense making about new concepts. This physical interaction creates the preconditions that support the emergence of perceptually based representations, and the eventual construction of abstract symbols, as physical and perceptual qualities are explicitly faded. Many educational approaches neglect this progression and instead follow the *formalisms first* approach to instruction (Nathan, 2012), wherein mathematical ideas are initially introduced in their most formal, symbolic, decontextualized form and only later grounded and applied. The conventional rationale is that the perceptual sparseness of abstract symbols benefits learners by reducing perceptual distraction (e.g., Kaminski et al., 2013). However, novices often flounder with early presentation of decontextualized symbols (Nathan, 2012). Experimental comparisons reveal benefits of concreteness-fading instruction over formalisms-first instruction for a wide range of mathematical concepts spanning elementary arithmetic, middle school and secondary level algebra, and postsecondary systems-theory concepts (Fyfe et al., 2014). Concreteness fading is especially well suited for fostering key STEM education principles in design- and product-oriented collaboration, as commonly implemented in PBL settings.

2.3 Transfer: Mapping of Invariant Relations to Achieve Cohesion

From an embodied perspective, the crux of transfer is establishing cohesion across contexts and physical instantiations, such that modes of perceiving and acting appropriate for engaging with a mathematical relation in one context (i.e., with a particular object or representation) also meaningfully apply in another context. In past work (Nathan et al., 2013; Nathan, Wolfgram, et al., 2017), we identified the significant challenges that students faced as they developed, to varying degrees, the skills for noticing and acting on similarities of different materials, labels, ecological contexts, iconic representations, and symbolic notations by virtue of their shared invariant mathematical relations. For example, in the excerpt of an engineering lesson presented at the outset of this chapter, the notion of *angle of ascent* (THETA) is depicted in a variety of *modal forms,* including speech, symbols, gestures, and diagrams (and, later, in a working physical device); however, a single invariant mathematical relation underlies all of these forms. To establish cohesion, various modal forms must be regarded by students as similar in terms of their perceptions and actions.

Critically, these differing modal forms vary in the actions they afford. Following J. J. Gibson (1966, 1979/2014), we define an *affordance* as the complementary relation between an object (which we take to include symbolic and material objects that are physical or imagined) and an actor who engages with that object. In an engineering lesson, for example, a physical device may afford grasping and holding, whereas the symbolic expression that mathematically models the behavior of that device does not. Thus, the processes of perceiving and acting that apply to one modal form may not apply to another modal form. For cohesion to be produced, the perceptions and actions applied to one modal form as it is manifest in one context must evoke in the actor a connection to a related modal form, which may be encountered in the same or in a different context.

A striking example of cohesion production is provided by Alibali and Nathan (2007) in an early algebra lesson for sixth-grade students. The teacher sought to connect a drawing of a pan balance scale (the initial modal form) that had an arrangement of blocks placed on the two sides to a symbolic equation (the second modal form) that represented that configuration of the balance scale with literal symbols and arithmetic operators. In the first such arrangement, two spheres on the left pan exactly balanced the sphere and two cylindrical blocks on the right pan.

The teacher emphasized that simultaneously removing the same type of block from the two sides of the balance scale corresponded to subtracting the same value from both sides of the equation, thus establishing the mapping between the balance scale and the equation and extending the original action that applies to the pan balance (object removal) to algebraic manipulation (symbolic subtraction). This provides a clear example of how embodied processes support transfer by depicting the ways these lifting actions can be applied first in a primary context (pan balance) to a second context (symbolic equation). It also shows how a teacher simulates the lifting of two literal symbols simultaneously from each side of the equation as a way to maintain cohesion when shifting modal forms from objects on a balance scale to an equation.

For a learner to exhibit transfer of knowledge across different contexts, a *mapping* between the actions afforded by the modal forms in each context must be made to establish cohesion. Mapping may be spontaneous or require instructional support. Lobato and colleagues (e.g., Lobato, 2003; Lobato, Ellis, & Muñoz, 2003) highlight ways the educational environment can be structured to orient learners' attention to such mappings, and they refer to such practices as *focusing phenomena*.

Evidence that a mapping has been formed may then be revealed in learners' later behaviors. For example, we may observe students tilting a ballistic device (e.g., a catapult) to launch a projectile at a particular angle in a way that is fundamentally similar to solving the range equation for a particular value of THETA. That is, the device acts as a "range function" that "computes" the landing distance of an object given (virtually) any input angle, which is achieved by tilting the launch pad. We consider evidence of such a mapping later in this chapter.

In brief, we argue that transfer occurs when learners and teachers establish cohesion of their experiences by mapping modes of perceiving and acting that they successfully used in one context to a new context. Learners and teachers *express* that cohesion across contexts in a variety of ways, principally through speech, gestures, and actions, including simulated actions.

Note that mapping supports identification of invariant relations by juxtaposing contexts that afford corresponding modes of perceiving and acting. Importantly, this identification and mapping may be implicit or explicit for the learner. This view of transfer differs from classical theories that rely on extracting common knowledge structures or rules with generalized conditions for application.

Transfer, by this account, centers on two distinct but related processes: *constructing a mapping of an invariant relation across contextualized modal forms* and *expressing cohesion* established by that mapping, as indicated by various behaviors, as described below. We consider each of these processes in turn.

2.3.1 Mapping as a Mechanism for Cohesion

We posit that mapping is a mechanism for establishing cohesion. Mapping can be aided by the focusing "moves" made by teachers, parents, curriculum designers, and knowledgeable others who already apprehend connections, and it can be supported by contextual cues, such as spatial alignment, labeling, and deictic gestures. Mapping can also be managed by learners who regulate their own environments to provide helpful contextual supports, such as placing information side by side.

Mapping involves constructing a relation between two (or more) objects, inscriptions, or ideas. We argue that there are multiple mechanisms by which mapping may occur. In some cases, learners may engage in explicit analogical mapping. For example, a child might reason about fraction division by explicitly mapping elements of a given fraction-division problem to elements in a whole-number division problem, saying, "6 ÷ ¼. Well, if I was doing 6 divided by 2, I would make groups of 2. So, 6 ÷ ¼, I'm going to make groups of ¼." In other cases, learners may perform mapping in a more implicit way, via *relational priming*, a process by which exposure to some task or situation primes a relation that can then be recognized or used in a novel task or situation (Day & Goldstone, 2011; Leech, Mareschal, & Cooper, 2008; Sidney & Thompson, 2019). For example, after modeling whole-number division problems with cubes— by forming groups the size of the divisor—a learner might enact the same relation to model a fraction-division problem because that relation (forming groups) was primed in the initial task (Sidney & Alibali, 2017). Another means of forming the mapping is through *conceptual metaphor* (Lakoff & Núñez, 2000), where one idea, such as arithmetic, is referred to in terms of another idea, such as object collection. As in this example, the second domain (the *target domain*) is more familiar and more concrete than the first, *source domain*. Conceptual metaphors are grounding, inference-preserving cross-domain mappings. Using conceptual metaphor, the inferential structure of one conceptual domain (say, whole numbers) is used to reason about another (say, fractions). In still other cases, learners may map relations via *conceptual blending* (Fauconnier & Turner, 1998, 2008), a mechanism by which people link two ideas that share structure, and "project selectively from those inputs into a novel 'blended' mental space, which then dynamically develops emergent structure" (Fauconnier, 2000, p. 2495). All of these forms of mapping—analogical mapping, relational priming, conceptual metaphors, and conceptual blends—forge correspondences, and these correspondences may afford engaging in corresponding modes of perceiving and acting.

Because transfer involves mapping modes of perceiving and acting from one context or representation to another to produce cohesion, we assert that pedagogical moves that support mapping are integral to transfer. Indeed, teachers engage in many practices, both planned and spontaneous (Alibali et al., 2014; Nathan, Wolfgram, et al., 2017), that highlight invariant relations across contexts, representations, and material forms. In subsequent sections, we highlight several distinct mapping practices that teachers use, both in ordinary mathematics instruction and in PBL settings, including *projecting* invariant relations across time and space and *coordinating* representations using techniques such as consistent labeling, linking gestures, and gestural catchments (Nathan et al., 2013).

2.3.2 Expression of the Mapping

If, indeed, this mapping of modal-specific ways of perceiving and acting is at the heart of transfer, it will be expressed—at least in some cases—in learners' behaviors. Learners may, for example, appropriate actions or ways of thinking applied in one context for use in another, and they may make mappings (either implicit or explicit) between the contexts. Some aspects of learners' behaviors in the novel context—their language, gestures, or actions—may reveal the mapping of modal-specific forms of perceiving and acting from a prior context (Donovan et al., 2014).

Learners' behaviors in different contexts often involve different sorts of actions, and their gestures in novel contexts may reveal activation of action patterns that they have produced in other contexts (Donovan et al., 2014). Learners may produce gestures in novel contexts that are similar in form to actions they produced in previous contexts. This repetition of gesture form—termed a gestural "catchment" by McNeill (2000)—is thought to reveal cohesion in speakers' thinking. Gestural catchments may reveal implicit or explicit mappings between contexts, representations, or material forms (Donovan, Brown, & Alibali, 2021).

Mapping often involves forming a conceptual blend, and such blends can be expressed in many ways (Fauconnier & Turner, 2008; Williams, 2008). When conceptual blends are established in classroom settings, the physical context typically offers a material anchor for the blend. Thus, the blends observed in PBL settings are often *grounded blends* (Liddell, 1998) that include elements of the immediate, physical environment. For example, a student may mount a protractor on a catapult arm and rewrite the angular measures as distances to the target, thus using a material anchor to blend angular measure with projectile motion using trigonometry and the laws of kinematics. The actions that the student previously applied to the original artifact (such as adjusting the angle of the protractor) can support new, inferential actions, such as retargeting based on lineal measure, which extend the student's repertoire of actions into the space of the new conceptual blend (Williams, 2008).

The earlier example of a teacher simulating the lifting of the same symbols off two sides of an equation, much as one lifts the same objects off two sides of a balance scale, is one such conceptual blend. Here, we can see how the mapping is formed. In this conceptual blend, the equation is treated as a pan balance and the adding and removing of objects to maintain balance maps to the manipulation of terms in the equation to maintain equivalence. Further, the teacher expressed this mapping explicitly in speech, noting that she wanted to "take a sphere off of each side" but saying that "instead of taking it off the pans, I'm going to take it off this equation." Thus, she identified the invariant relation of maintaining equivalence, performed the mapping of the pan balance to the equation with an explicit verbal link, and expressed cohesion across the modal forms through the reapplication of gestures that depicted the same actions. This mapping is illustrated in Fig. 2.1.

Other features of the teacher's speech also manifest her effort to align the diagram and the equation. For example, she used the same pronoun to refer to the sphere pictured in the diagram and the symbol *s* in the equation: "Instead of taking *it* off the pans, I'm going to take *it* off this equation" (emphases added), thus highlighting that the two inscriptions refer to the same quantity. She also used the same verb—*taking off*—to refer to removing a sphere from each side of the pan balance and subtracting *s* from each side of the equation. Thus, she highlighted the correspondence of these actions using a common label.

The teacher also expressed the correspondence between the pan balance and the equation in her gestures. She used a grasping gesture with both hands to gesturally depict taking the blocks off the two sides of the scale—a simulated action (Hostetter & Alibali, 2008, 2019) over the drawing of the scale. She then produced this same grasping handshape over the corresponding symbols in the equation to refer to subtracting values from the two sides of the equation. With this gestural catchment, the teacher sought to communicate the invariant relation of equivalence as "remove the

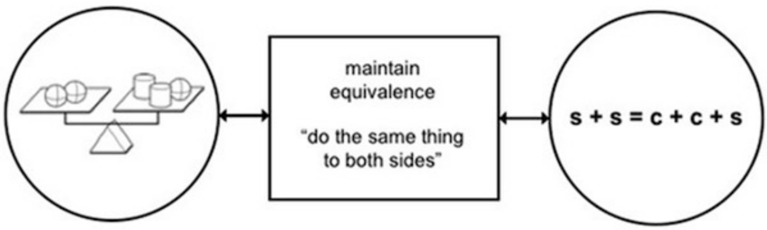


Fig. 2.1 The math teacher identified an invariant relation of maintaining equivalence and performed the mapping of the pan balance to the equation with an explicit verbal link and repeated gestures that depicted actions

same quantity from both sides" as it applied both to the physical pan balance depicted in the drawing and to the symbolic equation.

Note that this teacher *simulated* the action of "grasping objects" over both the diagrammatic and the symbolic representations, even though neither of these two inscriptions (diagram and equation) would afford this physical action. Both are two-dimensional representations, so their elements cannot be grasped or picked up. Importantly, however, the teacher's hands were configured as if actually grasping objects, and, in this way, her gesture evoked the physical objects that were represented symbolically in the diagram and the equation. Thus, in this simulated action, the teacher expressed a set of analogical relationships among the physical situation—which would afford such action—and the two inscriptions.

Thus, this conceptual blend was expressed in a range of ways: via an explicit verbal link, via common labels for related elements, and via a gestural catchment of the same simulated action performed in both spaces. The blend was grounded both in the two inscriptions, which were physically present, and in the (absent) physical objects that were evoked by the configuration and motion of the teacher's hands in real space (cf. Liddell, 1998). Using speech and gestures in these ways, the teacher organized corresponding elements of different representations with reference to one another, linking them together multimodally, in an effort to help students apprehend their connections.

This example also illustrates the centrality of the teacher in our theory of transfer. Teachers use a range of verbal and gestural techniques to support students in identifying the invariant relations and making the relevant mappings across contexts, representations, and material forms to establish cohesion (Alibali et al., 2014; Nathan, Wolfgram, et al., 2017). This is why we claim that the pedagogically designed actions of teachers—as well as parents, collaborators, and curriculum developers—are an integral part of transfer when viewed from an embodied perspective. We further suggest that expressing cohesion in the various ways described here is productive for learners' thinking, in the sense that it affirms, strengthens, and reifies the mappings across modal forms that have been established. It also serves as an effective means of communicating these mappings to others during collaboration or instruction.

2.4 Illustrating Embodied Transfer in a PBL Context

In this section, we provide examples from a PBL engineering classroom that demonstrate the power of an embodied theory of transfer to account for both successful and unsuccessful transfer. The examples also illustrate how a teacher's pedagogical moves foster cohesion for students in the PBL classroom and are thus a necessary part of an embodied account of transfer. The examples show how successful transfer arises by establishing this cohesion, whereas unsuccessful transfer occurs when learners' actions remain overly restricted to earlier modes of perceiving and acting.

2.4.1 The Three Central Elements when Analyzing Transfer from an Embodied Perspective

The accompanying examples illustrate the complex process of transfer that students and teachers face in the PBL classroom. Photo Transcript 1 (Table 2.2, which includes the excerpt from the chapter introduction) is taken from an engineering class in a U.S. Midwestern urban high school in late spring, near the end of the school year. This excerpt sets the PBL design challenge to build a ballistic device that can make a projectile hit a basket at some location, undisclosed until the last moment, with successful engineering based on the underlying math and physics of projectile motion. Even the open lecture, which focuses on trigonometry and kinematics, is rich with embodied methods of grounding the target invariant relation and other associated mathematical ideas and helping to foster cohesion as these ideas are manifest in multiple modalities, including symbols, drawings, words, wood, and the teacher's gestures.

We distinguish between the authentic classroom learning experience in which the students and teacher are embedded and the analytic process that is undertaken by researchers who study these classroom events. In terms of analysis, there are three central elements of transfer. First, it is critical for the analyst to *identify the invariant relation* that is central to the curriculum design and threaded throughout the modal forms. For this multiday unit, for example, the invariant is THETA, highlighted by the teacher on Day 1 and labelled as the "angle of projection." Second, the analyst must *describe the mapping* of the invariant relation across the range of modal forms used in the series of lessons. Third, the analyst must be able to describe how this mapping is expressed by the teacher and the students in the learning environment.

Separately from the analytic concerns of researchers, for learners to experience a sense of cohesion across the various modal forms and contexts that are the hallmark of the project-based curriculum, they must construct for themselves the mapping of modes of perceiving and acting that, optimistically, will apply across contexts. The mappings that are part of the expert model of transfer are important for the curriculum design, and may be shared in teacher supplementary materials, but they often remain implicit to the students (Prevost et al., 2014). Learners act on the new modal forms (e.g., their design sketches, mathematical models, and machined devices) in accordance with their constructed mappings. Learners' actions may operate in accordance with the expert model, indicating effective near and far transfer, as will be seen in Photo Transcript 2 (Table 2.3). Alternatively, learners' actions may be applied to subsequent modal forms in ways that do not align with the conceptual structure of the invariant relation, leading to "false transfer," as illustrated in Photo Transcript 3 (Table 2.4).

In the examples that follow, THETA is most commonly invoked by the teacher and by several of the students in a gesture of a flat hand posed at a fixed angle or of a flat hand pivoted at the wrist to refer to the range of angular values that THETA can take. The repeated expression of this idea in gesture makes up a gestural catchment, which reinforces cohesion across different manifestations of the invariant relation. Further, THETA is also evident in the design sketches created by the students and in the material devices that students build as they strive to create a ballistic device that can be adjusted "on demand" to enable a projectile to precisely hit a desired target.

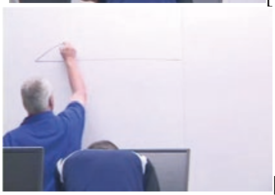
An invariant relation across modal contexts This first photo transcript demonstrates (a) identification of the invariant relation and (b) the ways a teacher uses pedagogical actions to highlight for students the mapping of the invariant relation across multiple modal forms.

Table 2.2 Photo Transcript 1: Day 1

Line	Transcript	Photo
1	T: I had given you an assignment to start working on a ballistic device that will throw a ping pong ball.	
2	T: And we had some constraints with that, um on a handout that I gave you. Particular constraints.	
3	T: What I wanna do today, is talk about, **the angle of the projection [1]**, that we shoot this, fire our ping pong ball and the **distance [2]** it'll go.	 [1]  [2]
4	T: And kinda mathematically determine what's the **best angle [3]** to get the maximum range, given a set velocity, of that we're firing this thing, okay?	 [3]
5	T: So we know that we can change the distance.	
6	T: What are some of the ways that we can change the distance, if we're shootin' a ping pong ball out of a device? [Name]?	
7	S: Angle of like, the ball.	
8	T: Okay. **Angle of projection. [4]**	 [4]

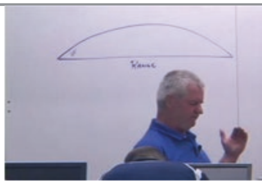
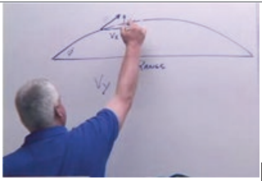
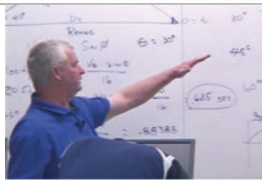
(continued)

Table 2.2 (continued)

Line	Transcript	Photo
9	T: That's gonna have an effect on it, right? What else?	
10	S: Velocity.	
11	T: Velocity. Which is, the speed in a certain, **in a set direction [5]** that we wanna go, 'kay.	 [5]
12	T: Those basically are the two elements that are gonna **affect the range [6].**	 [6]
	[Omitted portion]	
13	T: Alright so up here on the board, I want you to follow along, this is definitely a little bit complicated but I think we can get a handle on it.	
14	T: We're gonna—we're gonna look at two aspects of this.	
15	T: One, we're gonna look at the angle that affects our range.	
16	T: And once we pick, a-a-and then after we select an angle, we're also gonna calculate the range that we can get by, with those different angles.	
17	T: So let's look at how this works.	
18	T: First of all, put this over here, so draw it along with me.	
19	T: What happens when the, when we **project something through the air [7]**, is we end up with **something like [8] this** depending upon the angle here, which is *theta*.	 [7]  [8]
20	T: And, this is our range.	

(continued)

Table 2.2 (continued)

Line	Transcript	Photo	
21	T: And basically, what we have, is, **we're working with vectors here [9]**.		[9]
22	T: So we end up with, **some vectors that look like this and we call this, vector V_x and V, V_y[10].**		[10]
23	T: And we can say, that, V_y we're gonna start with V_y here.		
24	T: This distance this, right here.		
25	T: So we're gonna start with, V_y, equals V_o, sine, of *theta*.		
26	S: Mr. [Name], what's V_o?		
27	T: Actually V_o is going to be the velocity. 'Kay. Good question.		
	[Omitted portion]		
28	T: 'Kay, now to relate this to our project, I'm actually gonna give you a distance and I'm gonna say "okay we're gonna send, we're gonna set the basket fifteen feet away,"		
29	T: but whatever distance that is, I'm gonna decide that at the time.		
30	T: We're gonna set the, the basket so many feet away and you have to try to hit it.		
31	T: So by doing some calculations on, what you're, um, ballistic device fires, you can kinda set your angle hopefully to get, to get that distance.		
	[Omitted portion]		
32	T: Well what I want you to do is after you, **assemble your ballistic device, I actually want you to be able to gauge these angles on the device [11]**		[11]
33	T: and maybe we can stick an angle gauge in there somehow to check these angles		

(continued)

Table 2.2 (continued)

Line	Transcript	Photo
34	T: and you **determine at thirty degrees [12] what's your distance look like**.	 [12]
35	T: **At forty-five degrees [13] what's your distance look like [14].**	 [13]  [14]
36	T: At s-, at our range and at sixty, you know and so forth, get an idea of what your range is	
37	T: so that morning when we go down to the gym and we set this up and I throw a number at you	
38	T: which will be, it'll be somewhere between ten and twenty.	
39	T: So you're gonna have to try to design, you're gonna have to design your device to be able to fit within that parameter, constraints.	

We now analyze how the conditions for transfer are established by the teacher in this setting through his pedagogical actions. Our analysis of transfer in PBL settings rests on three analytic actions: (a) identify the invariant relation; (b) describe one or more mappings; and (c) document how participants in the learning environment express those mappings. Photo Transcript 1 illustrates the first two of these, with the mapping as a conceptual blend. The third component—how both the students and the teacher express those mappings using language, gesture, and action—is illustrated in Photo Transcripts 2 and 3.

The invariant relation is called out by the teacher as part of his presentation in Photo Transcript 1, Line 3, "talk about, the angle of the projection that we shoot this, fire our ping pong ball and the distance it'll go." Later, the angle of projection is referred to as "theta" by the teacher.

Describing the mapping of THETA involves identifying the relations among its various manifestations such that these seemingly dissimilar manifestations can be perceived as similar (Lobato, 2003). Our analysis reveals seven manifestations in all:

- as the measure of the sweep of an arm and hand to depict sample angular values (Line 3; photo [1]);
- as a drawn angle where the arc of the projectile meets the ground or baseline elevation (Line 19; photo [8]);
- as a Greek symbol (Line 21), called "theta," first written as the Greek letter Phi (φ) (photo [9]) and then later written as the Greek letter θ (photo [11]);
- in drawings and gestures that specify THETA as the direction of V_o, the initial velocity vector of the projectile that is related trigonometrically to component vectors V_x and V_y (Lines 21–23, photo [10]);
- as an equation parameter for computing velocity and range (Lines 23–25);
- as a physically manipulable quantity on the device students build ("you can kinda set your angle hopefully to get, to get that distance"; Line 31),
- as the reading from an angular measurement instrument (e.g., protractor; "I actually want you to be able to gauge these angles on the device and maybe we can stick an angle gauge in there somehow to check these angles"; Lines 32–34, photos [11] and [12]).

The intended result is a conceptual blend in which the manifestations of THETA are linked to one another in a cohesive network. Figure 2.2 presents a snapshot from the classroom depicting this network structure for THETA that, at that point in the lesson, is manifest in trigonometric relations, kinematics equations and diagrams, and gestures. Figure 2.3 illustrates the network of modal forms of THETA used throughout the unit.

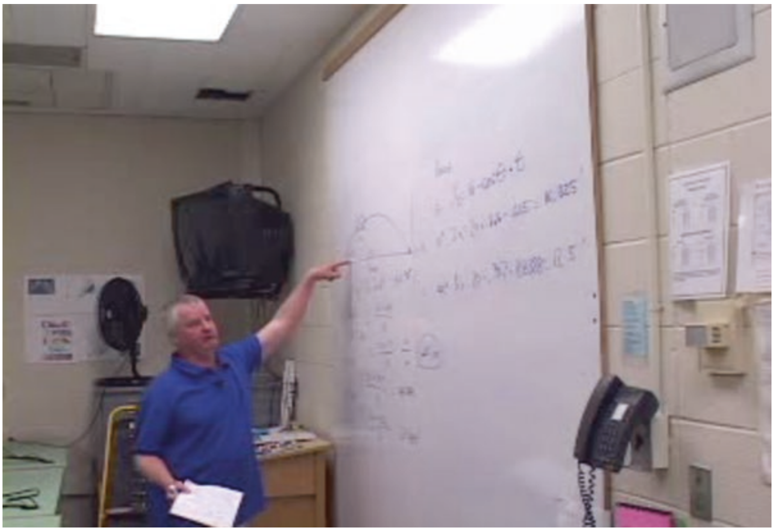


Fig. 2.2 Image of the whiteboard showing different manifestations of THETA

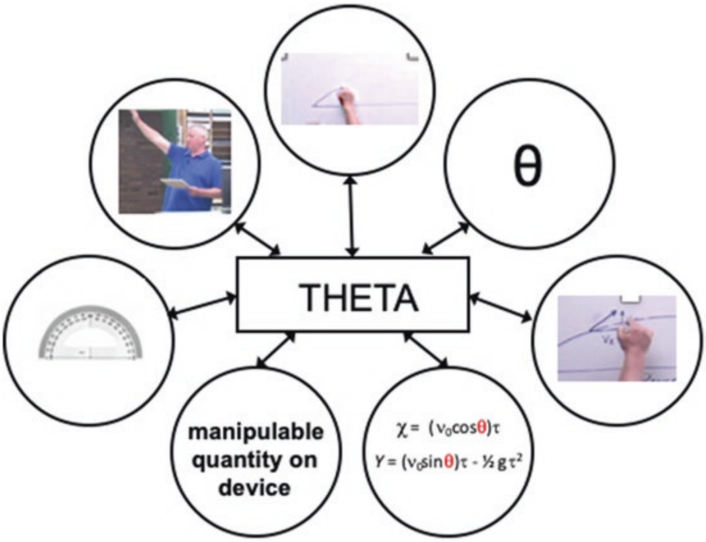


Fig. 2.3 The network of modal forms of THETA used throughout the unit

Successful transfer exhibited by students via gestural catchment The third element of analyzing transfer within an embodied framework is explicating ways those in the learning environment express cohesion. One expression of cohesion is illustrated in Photo Transcript 2 (Table 2.3), in which the teacher interacts directly with students who have been working in project design teams. To foreshadow, Photo Transcript 2 shows that at least two of the students express the cohesion of the invariant relation across two different instructional contexts: the formal lecture on kinematics given by the teacher, which involves a whole-class participation structure, and interactions that take place in the machine shop setting, which involve a small-group participation structure, which is the focus of the transcript. Here we observe the ways in which participants use body-based resources in several ways: to express the mathematical role of THETA that was depicted in the lecture; as it was drawn in their design sketch; as a measured and variable quantity; and in terms of its functional role for the project, which aims to control the trajectory of the projectile.

At the beginning of Photo Transcript 2, we observe two students (talking over one another) in a group of four express to the teacher how the design sketch they have drawn provides adjustments to the angle of projection (which they call at points “the elevation” and “different angles”) and a way to fix the angle of projection.

Student 1 notes (Line 8, photos [1] and [2]), “That’ll allow you to unscrew it, move it up and down,” and Student 2 concurs (Line 9). Especially notable is the gesture produced by Student 1 as he describes “move it up and down.” This gesture imitates the hand movement that the teacher previously used during the lecture to designate the many values THETA can take on, thus forming a gestural catchment.

Table 2.3 Photo Transcript 2: Day 1

Line	Transcript	Photo
1	T: Let's check, you guys. Where are you at?	
2	T: [Name] and [Name], what do we have here?	
3	S1: We got a, uh, thingy that works.	
4	T: Explain what you have goin' on here.	
5	T: 'Kay, so that is, where's your sheet with your constraints on it?	
	[Omitted portion]	
6	S1: Just a piece of wood to hold onto it.	
7	S1: Locking screw right there.	
8	S1: That'll allow you to **unscrew it, move it up and down (performs gesture three times in quick succession) [1] [2].**	 [1]  [2]
9	S2: (At the same time) Yeah.	
10	S1: And then **tighten it at whatever elevation you want [3]**.	 [3]
11	S2: **Different, different angles [4].**	 [4]
12	S1: A protractor sitting here. With a string with a weight on it.	
13	S1: So as you tip it, it'll, that'll tell you what degree you're tipping it.	
14	T: (At the same time) Oh! I like that. That's nice.	
15	S1: So that **tells you what degree so we can figure that out [5]**.	 [5]

The student presents an upright (left) hand with flat palm and proceeds to bend at the wrist and bring the hand back to upright three times in a couple of seconds, each time maintaining a somewhat flat palm. We regard this catchment as evidence that this student apprehends how THETA is manifest in the design sketch and that it aligns with the teacher's description.

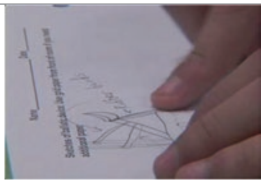
Student 1 continues, "And then tighten it at whatever elevation you want" (Line 10, photo [3]). He depicts this action by moving his right hand up to be near the left and making a motion typical for tightening a screw. Student 2 further immediately elaborates, "Different angles" (Line 11, photo [4]). In so doing, he, too, makes a gesture for the angle THETA twice in quick succession. This gesture repeats the gesture produced by the teacher during lecture and the gesture produced by Student 1 moments earlier, thus continuing to build the gestural catchment and providing further evidence that Student 2 also constructed a cohesive account of THETA as it relates to their design. The students further demonstrate their understanding as reflected in their method of measuring the angle of projection with the clever use of a weighted string moving across a protractor that is mounted on the device (Lines 12–15, photo [5]).

In brief, Photo Transcript 2 demonstrates how students express cohesion in this PBL activity through a gestural catchment and through connecting language directed at their design sketch (which provides a material anchor of one manifestation of THETA), the mathematics of THETA, and the angular measurement device. This excerpt also illustrates that the teacher contributes to transfer by using brief but important prompts. But it is the activity structure as a whole that really provides the mapping of the invariant relation across contexts by forging connections between the hands-on design project and the mathematics and physics presentation.

Unsuccessful transfer as inappropriate mapping of the invariant relation In contrast to Photo Transcript 2, which illustrates successful transfer, Photo Transcript 3 (Table 2.4) involves students who latch onto the wrong adjustable feature, so their design varies the initial velocity but not the angle of projection. The students' expressions of the mapping reveal this to be their constructed understanding, rather than a process of directly perceiving the invariant relation as labelled by the teacher. During this excerpt, the teacher recognizes that the students' actions reveal that their thinking and design is based on the incorrect mapping of the angle THETA to their device, which is contributing to unsuccessful transfer. In response, the teacher attempts to repair the mapping by reinstating the gestural catchment and making an explicit, direct mapping between the part of the device that could instantiate THETA and the mathematical inscriptions that model the influence of THETA on projectile motion that were previously written on the board.

The exchange in Photo Transcript 3 shows how transfer can be thwarted when students construct an inappropriate mapping for the target invariant relation. The teacher provides a rich prompt (Line 1), asking, "How are you going to change the angle of your trajectory?" invoking the gestural catchment that has come to signify THETA (photo [1]). The students have designed a catapult that includes rubber bands that can be set at different points before their release, altering the tension and therefore the speed with which the catapult arm will release. The students see the

Table 2.4 Photo Transcript 3 (Unsuccessful Transfer): Day 2

Line	Transcript	Photo
1	T: Alright now let me ask a question regarding **how are you going to change the angle of your trajectory [1]?**	 [1]
	[Omitted portion]	
2	S2: Right there.	
3	S1: We'll have this rubber band here, pull it down here.	
4	S1: And so we have several spokes here so the further we pull it down and attach it, that, **that changes the angle for us [2].**	 [2]
5	T: Well I'm wondering if the further you, pull your rubber band down–	
6	S1: Mmhm.	
7	T: –is gonna affect your, **velocity, more than your angle [3].**	 [3]
8	S2: [At the same time] Yeah it's, well no, this is the velocity	
9	S2: but what we're sayin' is that this is **how hard it pulls, but then right here [4]**, where it, where it, **where the fulcrum is like this actually you can tilt it [5].**	 [4]  [5]
10	S2: [At the same time] The rubber bands control the tension but the placement is what really controls...	
11	S2: Like. See what we're saying?	

(continued)

Table 2.4 (continued)

Line	Transcript	Photo
12	T: So it's, it, okay so, **if I could, suggest, I think that [6]**, you might be able to adjust your angle by, by having some type, by controlling where this stops.	 [6]
13	S1: Yeah.	
14	T: But that's probably also gonna affect your, maybe affect your velocity.	
15	T: What I'm saying is, either that or else you have to tip the whole thing.	
16	S2: No, we don't.	
17	S2: That's why, 'cause the two sides stay put but then the top part can, tilt, right there.	
18	T: Okay.	
19	S2: [At the same time] So the fulcrum can change positions, basically.	
20	T: Alright. So I think maybe what you need to do is is, take into consideration what I just said about–	
21	S1: Yeah.	
22	T: –**being able to control the angle [7].**	 [7]
23	T: **That's why we did everything we did here [8]–**	 [8]
24	S1: Mmhm.	
25	T: –with the math. Because we wanna–	
26	S1: (At the same time) The math yeah.	
27	T: –be able to adjust the angle of the trajectory.	
28	T: I would try to keep, the velocity, the same, consistent, throughout the whole, every test that you do that that's consistent	

(continued)

Table 2.4 (continued)

Line	Transcript	Photo
29	T: and so all you're gonna change once you, one you decide what that velocity has to be, **all you're gonna change is your angle [9].**	 [9]
30	S1: Yeah.	
31	T: Okay?	
32	S1: Mmhm.	
33	T: I don't really want you to use the tension on the rubber bands, as, the only control.	
34	**T: I want you to have an angle adjustment [10].**	 [10]

different positions of the rubber bands as taking different angles (see Fig. 2.4), which they predict will alter the angle of projection: "So the further we pull it down and attach it, that, that changes the angle for us" (Lines 3–4, photo [2]). In response, the teacher rightly observes (Line 5) that the catapult arm will release at the same angle regardless of the placement of the rubber band, but the change in tension will affect the initial speed of the projectile. The teacher points to the design sketch to help clarify his critique (photo [3]).

The students do not pick up on this critique but offer a defense (Lines 8–11), "See what we're saying?" This suggests that the students are not merely misinterpreting the theory or misreading their own design sketch. The second student speaker (Lines 8–9) offers this account, "Well, no, this is the velocity, but what we're sayin' is that this is how hard it pulls, but then right here, where it, where it, where the fulcrum is like this actually you can tilt it" and demonstrates this idea in photos [4] and [5].

A reasonable interpretation is that the students operate with a preexisting "ontological coherence" (Slotta & Chi, 2006) for velocity exclusively as a scalar measure of speed of the projectile, which interferes with their adoption of a new conceptualization of velocity as a vector quantity (i.e., V_o) that includes both speed and direction. Prior ontological commitments of this sort are notoriously difficult to alter. Here we observe such a case from two students in defense of their design when the first student says (Line 17), "That's why, 'cause the two sides stay put but then the top part can, tilt, right there," and the second (overlapping) says (Line 19), "So the fulcrum can change positions, basically." In neither case, however, will this design provide the control of the angle of projection that the project requires.

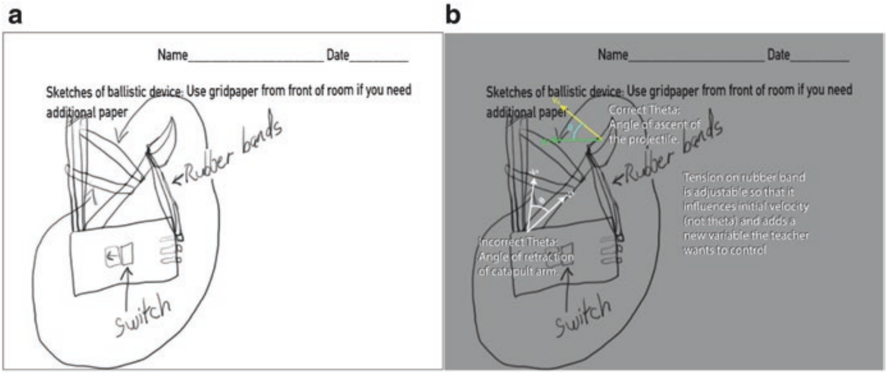


Fig. 2.4 (**a**) One group's original design sketch with (**b**) the vectors and angles added that label the correct and incorrect matches to THETA

An interesting part of this exchange comes when the teacher identifies the break in cohesion. By way of repair, he offers two mapping acts. First, he reinvokes the THETA gesture but this time does so in the same plane as the paper design sketch (photo [7]) while saying (Line 22) "being able to control the angle." In this way, the teacher connects the variation of the angle to the students' design sketch. Second, the teacher makes explicit reference, with speech, gesture, and upturned eye gaze (photo [8]), to the mathematical derivation still on the whiteboard at the front of the room and starts out saying (Lines 23–29), "that's why we did everything we did here with the math," and ends with, "all you're gonna change is your angle." The students acknowledge this midway and repeat (Line 26), "The math, yeah," but they seem disappointed by the teacher's reaction to their design.

2.5 Reflections on an Embodied Theory of Transfer

In this chapter, we have advanced the argument that transfer is fundamentally an embodied process. This is made especially evident when studying PBL settings. Learning and teaching in PBL settings are embedded in rich, multimodal contexts where content knowledge and information are often extended across a variety of semantic resources, including objects, inscriptions, and other actors. We assume that learners and teachers have a natural drive for cohesion in the learning experience—learners, to experience continuity, and teachers, to provide a meaningful and engaging learning environment in which their students achieve the desired understandings. We observe that both teachers and learners engage embodied processes as they map invariant relations across various modal forms. This mapping enables agents in educational settings to apply prior modes of perceiving and acting to new contexts and to create movements that will activate those invariant relations through transduction. Mapping may be explicit, as in analogical mapping; implicit, as in the case of priming relational structures; or some combination, as may be seen with conceptual blends. Teachers and students express cohesion by connecting different contexts and different modal forms via speech, actions, and gestures, as when a teacher simulates picking up symbols simultaneously from both sides of an equation or when a student invokes a gestural catchment to indicate how a structural property of a device enacts the relationship depicted in a mathematical model. We now consider some notable aspects of the proposed theory, implications for educational practice, and open research questions that may advance understanding of transfer.

We have argued that there are three core elements to embodied transfer: (a) identifying the central invariant relation that is manifest in multiple contexts, representations, or modalities; (b) mapping that relation across those contexts, representations, or modalities; and (c) expressing cohesion across the disparate manifestations of that invariant relation. We view the order of these three elements as somewhat fluid. Mapping across contexts—performed by a teacher, for example—might precede a student's awareness of the central invariant relation. The mapping can provide a means for comparison that enables the learner to perceive connections between contexts and inscriptions, as when students experience that they are performing similar actions in ontologically different contexts. The actions performed in the new context can activate common cognitive states through transduction, which then help the student to notice the invariant relation in the new context, thereby enabling mapping across the contexts. Expression can also play a role in making implicit mappings more explicit for the learner, as when students' reflections on their motoric behaviors bring these relations into conscious awareness. This may be one reason why self-explanation is a powerful mechanism for promoting transfer (see, e.g., Rittle-Johnson, 2006).

An important assumption of an embodied theory of transfer is that transfer operates within a predictive architecture and a set of feedforward mechanisms that ready the system to act. Consequently, transfer is not an occasional process but a continual one. A system always looking to act will also activate cognitive states in accord with its actions. This offers a theoretical basis for understanding near and far positive transfer as well as negative transfer. In this framework, near transfer is especially likely when modes of perceiving and acting from an earlier context are activated and readily apply in a new context. The teacher simulating lifting the drawn objects off of the drawn pan balance is one such case, given that these affordances for a physical pan balance would normally apply. We describe as *far transfer* those cases in which the earlier modes of perceiving and acting are not directly applicable and that require some modification and some enhanced mapping support to establish correspondences. Negative transfer is expected when the mapping is salient but the associated modes of perceiving and acting are no longer relevant. One example is the "add all the numbers" error commonly made by elementary and middle school students solving mathematical equivalence problems (e.g., offering "15" as a solution for a problem such as 3 + 4 + 5 = 3 + __; Knuth, Stephens, McNeil, & Alibali, 2006; McNeil, 2014; Perry, Church, & Goldin-Meadow, 1988).

Our proposal also raises the issue of *false transfer*, which may occur when actors apply modes of perceiving and acting that they expect to be applicable but that match only at a surface level and which therefore do not yield successful transfer (in terms of experts' expectations). The persistence of false transfer in the face of feedback may be due to students' prior ontological commitments that offer strong matches to the current circumstances (Chi, Roscoe, Slotta, Roy, & Chase, 2012; Slotta & Chi, 2006). One example is treating velocity as a scalar measure of speed in a design project that requires that velocity be treated as a vector quantity specifying both speed and direction. The activation of inappropriate modes of perceiving and acting can help explain why tasks that share surface structure but different invariant relations so readily lead to false transfer.

As these classroom examples make clear, transfer is an embedded process, situated in a particular physical and sociocultural learning context. PBL is also an extended process such that multiple actors (often a teacher and students) are engaged in transfer, mapping invariant relations across modal forms. The contributions of both teachers and learners to transfer suggest that transfer is a fundamentally social activity (Lobato, 2006). This view suggests several powerful ways to promote transfer, particularly in complex learning environments. In past work (Nathan, Wolfgram, et al., 2017), we documented some of the key processes that teachers draw on to foster cohesion across representations, contexts, and settings: Teachers actively bridge ecological shifts when learning takes place in different ecological contexts (such as the classroom and the machine shop), and teachers check that their students are aware of the continuity they strive for; teachers coordinate ideas across different spaces using common labels, thoughtful juxtaposition, gestural catchments, and deixis in both speech and gesture; and they project invariant relations forward and backward in time to promote temporal continuity. Our position is that these pedagogical processes are integral to transfer. Excluding the teacher from a theory of transfer risks creating a theory that is unable to account for transfer as it occurs in authentic settings.

Our theory also highlights the importance of understanding the fine structure of the ways in which teachers and students express cohesion. In this regard, we draw on Goodwin's (2013) observation that speakers commonly *layer* semiotic fields one upon another during discourse, a process he termed *lamination.* In our view, teachers and students may laminate different representations together—that is, layer them together in space or time using language, gesture, or action—thereby fusing them conceptually. For example, consider the teacher (described earlier) who produced the same gesture of removing objects from two sides over a drawing of a pan balance and then over a symbolic equation representing the state of the pan balance. With this catchment gesture, the teacher laminates together the pan balance and the equation. She organizes elements of these manifestations of the invariant relation with respect to one another and uses gestures to express their correspondences.

An embodied account of transfer can also provide insight into why certain instructional approaches have proven effective. The proposed theory naturally explains the success of instructional approaches that bring actions in target contexts into close alignment with actions in the original source context. For example, bridging instruction (Nathan, Stephens, Masarik, Alibali, & Koedinger, 2002) relies on mapping students' invented strategies for algebraic reasoning to those that experts have identified as important for achieving curricular goals. Concreteness fading (Fyfe et al., 2014) helps learners to ground formal notations in terms of familiar modes of perceiving and acting, applying the resulting actions to a broader range of contexts.

An embodied account of transfer also has implications for assessment practices. Regarding formative assessment, it is well documented that learners sometimes exhibit ways of thinking in actions and gestures even before they have explicit awareness of their new understanding or before they have constructed verbal accounts of their new ways of thinking (Church & Goldin-Meadow, 1986; Goldin-Meadow, Alibali, & Church, 1993). Teachers who notice these nonverbal expressions can more accurately model students' conceptual development and can be responsive with their own pedagogical actions. Even untrained adults generate more accurate descriptions of children's understandings when they attend to children's gestures along with their verbal utterances (Goldin-Meadow, Wein, & Chang, 1992). Improving teachers' skills for noticing students' gestures can greatly enhance teaching and learning (Roth, 2001).

Summative assessment is generally more evaluative, taking place at the end of a major curricular unit. Summative assessment practices are dominated by students' verbalizable knowledge, often excluding learners' embodied forms of expression and therefore underestimating student knowledge. Further, assessment methods using computer keyboards can interfere with body-based forms of expression and can even impair students' thinking (Nathan & Martinez, 2015).

An embodied account of transfer raises several important questions for future research. First, what kinds of discourse practices contribute to students' identification and mapping of invariant relations across contexts? For example, to what extent are instructional practices such as using common labels or producing gestural catchments valuable for supporting students' mapping across contexts? Relatedly, which discourse practices help learners progress from an implicit, action-based understanding of invariant relations to explicit, verbalizable knowledge?

Second, do effective approaches to mapping depend on the target concept or on the age, prior knowledge, or cognitive skill of the learner? It is possible that some learners may benefit from more explicit mapping, whereas others may do better with more implicit approaches. These individual differences, in turn, may be due to differences in learners' prior knowledge or in their patterns of cognitive skills.

Third, what are the consequences of variations in mapping practices or variations in expressing cohesion? For example, do some types of mapping lead to more durable knowledge or to greater gains in students' conceptual understanding of the target mathematical concepts? Does expressing cohesion in gestures or speech help learners to stabilize that knowledge and make it more explicit? These questions raise further issues about underlying mechanisms, which can be construed at a variety of different grain sizes. One potentially fruitful level of analysis involves considering the management of attention in social interactions that focus on transfer. How do teachers' mapping practices affect students' attention to aspects of the context or to features of the particular representations being linked? More generally, how do contextual supports and social guidance of transfer influence learners' attention, and how is attention involved in identifying invariant relations and in mapping across contexts?

Finally, given that our account has emphasized the social aspects of transfer, how do dimensions of social relationships, such as warmth, respect, and power, affect patterns of transfer? For example, are students especially likely to attend to novel mappings expressed by social partners who display respect for their ideas and concern for their learning (Gutiérrez, Brown, & Alibali, 2018)? How does the history of a social relationship affect the negotiation of transfer by individuals in that relationship?

Although there are many questions yet to be addressed, we believe that an embodied perspective yields a novel and valuable conceptualization of transfer. There is increasing awareness among both scholars and practitioners of the embodied nature of cognition (e.g., Barsalou, 2008; Glenberg, 1997; Rosenfeld, 2016; Wilson, 2002). In our view, an embodied perspective on transfer is necessary because transfer occurs in a rich physical and social world. By focusing on invariant relations, how they are mapped across contexts, and how cohesion across contexts and across modalities is expressed and negotiated, we open new avenues of inquiry, and these avenues promise to shed light on transfer as it occurs in PBL settings and other complex learning contexts.

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