Teaching teachers teaching students: How embodied cognition can help pre-service teachers assess students' mathematical thinking

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Abstract

K-12 pre-service teachers participated in an online embodied geometric curriculum using a motion-capture video game and design tool, The Hidden Village. Players performed mathematically related body-movements (i.e., directed actions) prior to determining if a given geometric conjecture was either always true or false. After gameplay, groups of participants collaborated to create their own directed actions representative of geometric objects and enactive of transformations. Combined, the gameplay and design activities allowed researchers to investigate how an embodied curriculum for teacher training could impact teachers' awareness of how gestures and movements can facilitate thinking mathematically, for both their students and themselves as a means of formative assessment practices.

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Objectives

In mathematics classrooms, students often communicate their thinking to teachers in the gestures they make when asking questions, explaining their thinking, and providing a rationale (Abrahamson et al., 2020). Despite physically grounded origins of mathematical thinking (Lakoff & Núñez, 2000), much of school-based mathematics curricula is still dominated by a-modally abstracted approaches (i.e., symbolic, diagrammatic) that tend to neglect the embodied nature of mathematics, especially the role of gesture (Nathan, 2021). This creates practical issues in instruction and assessment where many math teachers are unaware of the valuable information contained in students' non-verbal behaviors that express what they know and how they know it.

To investigate ways to support teachers' awareness of students' embodied mathematical knowledge, we developed an online research program that enables K-12 pre-service teachers to experience and reflect on their embodied geometric reasoning, in two parts: (1) through action-based video gameplay and (2) an embodied co-design activity. For gameplay activity, we used *The Hidden Village* (THV), a motion-capture video game in which players are guided to perform directed actions (i.e., upper-body movements) that are emblematic of geometric concepts before evaluating the truth of geometric conjectures (e.g., "*The opposite angles of two lines that cross are always the same*"). After gameplay, teachers collaborated in a co-design activity to create directed actions for new conjectures using the THV design tool.

These gameplay and co-design activities provided opportunities for pre-service teachers to understand how performing mathematically related movements and developing body-based actions can support students' geometric thinking. We hypothesized that these embodied learning activities will improve teachers' awareness of students' gestures during mathematical thinking and communication, as well as teachers' abilities to accurately assess students' geometric reasoning by interpreting their gestures.

Theoretical Background

Drawing from the theory of *Gesture as Simulated Action* (GSA; Hostetter & Alibali, 2018), studies have shown that mathematics can be learned through action-based interventions (Abrahamson & Sánchez-García, 2016; Smith et al., 2014). Since gestures activate perceptual-motor processes in the brain when co-articulated with speech or thought, these sensorimotor experiences induce cognitive states through a process that Nathan (2017) calls *Action-Cognition Transduction* (ACT).

Interventions that use directed actions provide a body-based way for learners to conceptualize some of the spatial dimensions, relationships, and transformations of geometric objects relevant for promoting mathematical reasoning. These ACT-based interventions (Nathan & Walkington, 2017) can become tools for teachers to help transform formalisms of instruction into action-based interventions that ground abstract concepts for students (Alibali & Nathan, 2007; Roth, 2001).

In this study, we explore whether an embodied learning intervention affects teachers' awareness of students' gesture use in mathematical thinking. Specifically, researchers investigated changes in teachers' interpretations of the gestures that students made while reasoning about geometric conjectures. Our main research question is: How do these embodied interventions affect teachers' awareness of students' usage of gestures and teachers' abilities to interpret students' gestures while assessing students' mathematical thinking?

Methods

Participants

We recruited K-12 pre-service teachers (N=16) enrolled in math teaching courses at a large midwestern research university in the US. Teachers were separated into four groups, two groups in each of two different grade bands, K-5 and 6-12. Participants received a $150 e-gift card. Participation took place entirely online using Zoom. Each participant's individual and collaborative audio and video were recorded.

Materials

The Hidden Village (THV). THV is an interactive, 3D motion-capture video game that delivers an augmented embodied geometry curriculum. During gameplay, each player emulates the directed actions performed by an in-game avatar (Figure 1) after which the player evaluates if a given geometry conjecture is either *false* or *always true*. An example conjecture is "The opposite angles of two lines that cross are always the same."

The Hidden Village Conjecture Editor (THV-CE). This module is used for creating new contents for the gaming portion of THV. Players can expand the game content by designing their own sets of movement-based directed actions for THV. In this study, we used THV-CE for a co-design activity. Groups of participants collaboratively co-designed mathematically relevant directed actions (i.e., generated 3 poses of the avatar for players to mimic movements; see Figure 2). Once designed, users can preview the sequence of directed actions as a fluid animation. Since the co-design activity was conducted in a virtual setting (in response to the Covid-19 pandemic), a researcher acted as a proxy to operate the editing tool under participants' directions.

Outcome measures. Before and after the intervention, teachers watched short 1-minute-long videos of a student providing reasoning as to why a certain geometric conjecture statement is either always true or false. For privacy, the original student videos were transcribed and re-enacted by an actor. In response to the videos, researchers conducted semi-structured pre- and post-interviews, prompting teachers to: (1) explain how the student interpreted the mathematical concept and assessing the student's mathematical understanding, (2) refer explicitly to evidence from the student video, and (3) speculate how their observations might be used as formative assessments for their classroom instruction.

Procedure

On the day of the intervention, participants took part in a 3.5 hour-long online session that contains a series of activities, including: (1) each teacher-participant individually observing and commenting on an online video of a student reasoning with speech and gestures about the veracity of a geometric conjecture (pre-intervention measure), (2) paired, online gameplay of THV with another teacher-participant, (3) co-design activity through a whole-group discussion in a group of four teacher-participants, and then (4) individually watching and commenting on two new videos of a student reasoning with speech and gestures about the veracity of a geometric conjecture (post-intervention measure).

Data sources and analysis

Discourse Coding

To investigate how the embodied interventions impacted teachers' awareness and abilities to interpret students' gestures, we first transcribed the speech and gestures of their pre- and post-intervention interviews. These multimodal individual and collaborative transcripts were used to conduct qualitative analyses.

ENA Discourse Model

The transcripts were also analyzed using *epistemic network analysis* (ENA; Shaffer et al., 2016), a discourse analysis technique for identifying and quantifying the connections among cognitive elements in a discussion (see Table 1). The data was segmented by a turn of talk and coded using an automated coding process (nCoder; Marquart et al., 2018) based on regular expression matching techniques. All six emergent codes were validated using comparisons between a human rater and nCoder and pairwise Cohen's kappa scores ranged between $0.90 \leq \kappa \leq 0.98$ and Shaffer's rho values $\rho < 0.05$ (Shaffer, 2017).

ENA builds dynamic models of discourse as a nodal network and then calculates a mean centroid around which the discussion centers, weighting the connections between codes (Shaffer, 2017). ENA codes correspond to the epistemic elements that characterize a discourse. The edges reflect the relative frequency of co-occurrence between two codes. To test for differences between the networks of pre- and post-interview, we applied a two-tailed paired-sample t-test, assuming unequal variance to the location of points in the projected ENA space, then used the corresponding network graphs to interpret any statistically significant differences.

Results

Researchers used the multimodal discourse data from the pre-service teachers' pre- and post-intervention interviews to examine changes in teachers' awareness of students' gestures and teachers' abilities to interpret students' nonverbal mathematical reasoning. Across both pre- and post-intervention interviews, teachers noted students' gestures and used them as evidence to evaluate the student's mathematical understanding. However, there was a marked change from pre to post in the way teachers construed students' gestures while assessing their mathematical understanding.

Qualitative Results

Pre-Interviews. In pre-interviews, teachers frequently made hasty connections between the student's gestures and their mathematical knowledge. For example, when one student in the video made a static gesture relevant to the structure of the geometric conjecture, several teachers quickly concluded that the student had the correct idea, but they failed to fully account for *how* the student used the gesture in their reasoning process.

Figure 3 illustrates a pre-interview with teachers. This figure includes the screenshot photo of the re-enacted student (Panels A & B), images of Teacher 1 (Panels C & D) and a transcript of the teacher's speech and gestures while describing their interpretation of the student's mathematical understanding. Teacher 1's gesture in panel C emulates the student's specific gesture displaying vertical angles while panel D indicates that they recognize what the student's gesture represents. Teacher 1 makes a premature conclusion that the student has a basic understanding of the mathematical concept instead of considering the more precise function of gestures. Teacher 1's attention rapidly shifts to the student's utterance ("It either adds up to 180 or 360", panel B, lines 2-3) and deduces the student's level of understanding based on how the student expressed it without certainty (panel D in Figure 3).

For the most part, teachers' pre-interview interpretations of the gestures were limited, with teachers sharing more superficial impressions of the student's understanding, remarking at times on the student's attitudes and tone.

Post-Interviews. Teachers in post-interviews tended to pay more attention to the relation between the student's gestures and verbal utterances. For example, teachers were more likely to mention the role of the student's gestures while the student in the video was verbalizing their reasoning about the geometric conjecture.

In Figure 4, Teacher 2 (Panels B & C) determines that the student's justifications of their mathematical understanding (shown in Panel A) were insufficient based on the information from the student's speech and gestures while reasoning. Consequently, Teacher 2 focused on the core logic of the student's proof and justification ("The logic she was trying to use was almost like a contradiction, like a proof by contradiction I felt like"; see Transcript, lines 2 & 3). Next, Teacher 2 elucidates the ineffectiveness of the student's bent-hand gesture (panel A) in their reasoning process by saying "the gesture, she kept using, which is this [mimicking the student's poses by shifting from straight to bent-hand gesture in going from Panel B to Panel C; line 5], but like she didn't really do anything with it".

In this case, Teacher 2 considers the student's mathematical understanding not only as derived from the meaning of the student's co-speech gesture but also the contribution of the gesture in their overall reasoning process.

Quantitative Results

ENA of the discourse data from pre- and post-interviews showed significant differences in teachers' responses that corroborate the qualitative changes described. Figure 5 shows the mean epistemic networks for pre- (red network in upper left panel; $\text{mean}_{\text{Pre}} = -0.53$) and post-intervention (blue network in upper right panel; $\text{mean}_{\text{Post}} = 0.53$) as well as the subtraction of one from the other (lower panel). There is a statistically significant difference between the discourse patterns in pre-interview and in post-interview on the first dimension of ENA space ($t(26.93) = 4.15$, $p < 0.05$, Cohen's $d = 1.45$).

The mean subtracted network (lower panel of Figure 5) shows which connections account for the differences between pre- and post-interview. The *pre-interview* (red network) showed stronger links between EMBODIMENT and ASSESSMENT or EMBODIMENT and MATHEMATICAL THINKING. This shows teachers are initially preoccupied with using gestures to draw simple connections between student gestures and the quality of their mathematical reasoning. The *post-interview* (blue network) showed stronger connections between students' VERBALIZING and EMBODIMENT. This shows teachers gained increased awareness of mathematical reasoning as multimodal. These results corroborate the qualitative findings, namely, that teachers in pre-interviews were likely to make snap judgments of students' mathematical thinking (i.e., a limited interpretation of students' gestures), whereas teachers in post-interviews were more likely to integrate non-verbal information from students' gestures and speech as a more complete account of students' mathematical understanding.

Significance

This study demonstrated the potential of an embodied intervention to enhance teachers' perceptions, interpretations, and formative assessment practices. While these results are promising, there are some important limitations of this study. The small sample size limits the statistical power to substantiate current causal claims and will require replication. Additional analyses may provide greater clarity about the impact of the intervention on teachers' perceptions and interpretations of students' verbal and nonverbal ways of expressing their mathematical reasoning.

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Tables & Figures

Figure 1

Teachers performing directed actions for a conjecture during THV gameplay

Figure 2

Screenshot of teachers' discussion to collaboratively create new directed actions for conjectures during co-design activity

Table 1

Coding scheme

Code Name	Description	Example
MATHEMATICAL THINKING	Referring to describing mathematical concepts such as a triangle, conjecture, and opposite angles.	"The student was saying like 'they could need to be 180 or 360' and I was thinking that they were thinking of it as like supplementary angles."
EMBODIMENT	Referring to various forms of an embodiment, including gestures as well as speech describing an embodiment	"... [crossing arms vertically] using their hands and they referenced that opposite angles [pointing top and bottom part of the opposite angles], so the top and bottom angles are equal."
VERBALIZING	Referring to the teachers' pedagogical actions (e.g., review, evaluation, suggestion) based on students' verbal utterances	"... she said, 'boom boom' and I feel like that's not really a technical term to use when describing that kind of problem."
ASSESSMENT	Referring to the discussion regarding judgment on the level of students' understanding	"They have some sort of understanding of the concept, but it's not completely clear because they haven't fully demonstrated."
ALIGNMENT	Referring to the connection between gestures and speech	"The gestures didn't necessarily straight up aligned with what they were explaining."
MANNER	Referring to the attention to students' manner of talk	"I feel like she didn't have a good confidence in her explanation."

Note: Quotation indicates verbal utterances and brackets [...] indicate gestures

Figure 3

An example of the re-enacted student online video presented in Pre-Interview (panels A&B) and Teacher 1's response (panels C&D)

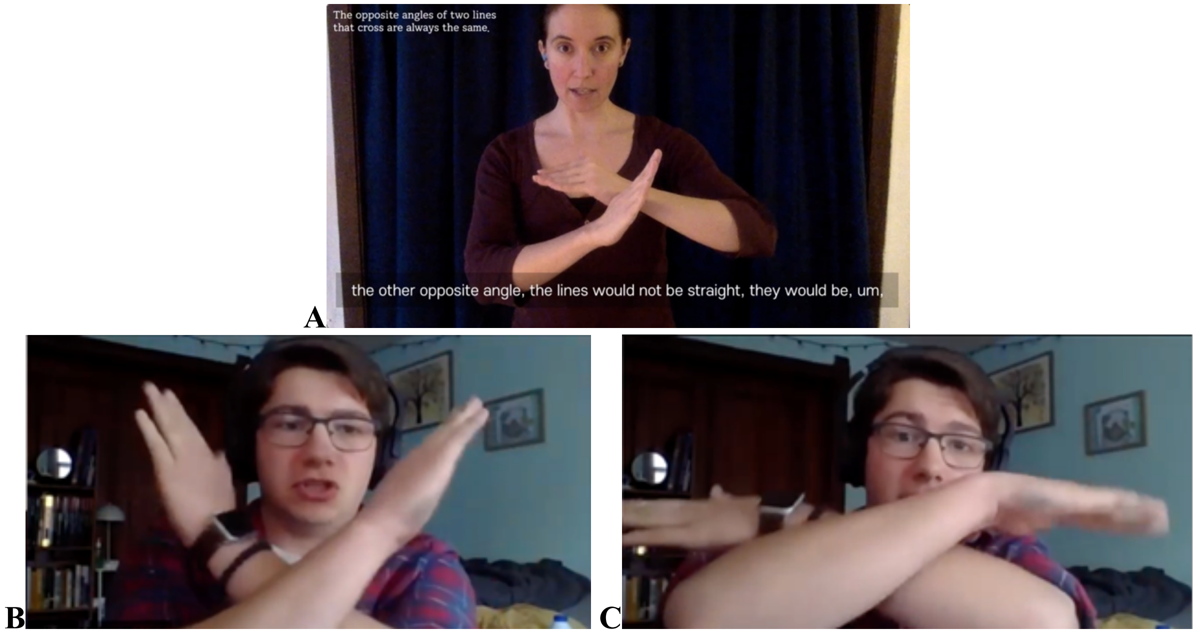
A B C D

Transcript

Teacher 1: [1] She understood the opposite rule [mimicking the student's X pose],
[2] but when she said "it either adds up to 180 or 360" and then was like
[3] I don't know really [making a pose to portrait 'I don't know'] at the end
[4] that just tells me that she doesn't fully understand the rule. Because if
[5] you understood the rule that would be the two side by side angles
[6] would equal 180, so she doesn't have the full understanding, but she
[7] has the very basic core understanding of what the rule should be.

Figure 4

An example of the re-enacted student online video presented in Post-Interview (panel A) and Teacher 2's response (panels B&C)



Transcript:

Teacher 2: [1] I thought she did a kind of poor job on that one. You very much could [2] see the problems. The logic she was trying to use was almost like a [3] contradiction, like a proof by contradiction I felt like. She was trying to [4] say "okay, well, if the lines aren't straight than the angles won't work". [5] And the gesture, she kept using which is this [mimicking the student's [6] sequence of poses], but like she didn't really do anything with it.

Figure 5

Mean ENA network diagrams showing the connections made in pre-interview (red, left) and post-interview (blue, right), and mean subtracted network (bottom).