When Conceptualization Gets Moving: Exploring How Directed Actions Complement Gestural Insights for Generating Geometric Reasoning

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Abstract

We document the phenomenon of gestural insights, which reveal ways nonverbal expressions exhibit students' mathematical reasoning. Thoughts conveyed through spontaneous gestures often differ from thoughts conveyed solely through speech. Undergraduate students performed directed actions, conceptually relevant body movements designed to foster conceptual understanding, before evaluating a geometric conjecture. Subsequently, students expressed gestural insights (physical expressions of correct conceptual understanding) with their bodies even when they were unable to verbalize their mathematical insights. These mismatches are the focus of two case studies that demonstrate how students recruit body-based resources to support conceptual thinking. We invite educators to consider the benefits of curating curricula and assessments that allow students to express knowledge and conceptual understandings through nonverbal and verbal means.

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The central objective of this paper is to describe two cases where students exhibit mathematical insights about geometric proof reasoning through gestures that are otherwise absent in their speech. We view mathematical proof as a multimodal discourse practice (Gresalfi & Cobb, 2006). When exploring geometric proofs, teachers and students often communicate key mathematical ideas using descriptive language, verbal inference, and gestures (Alibali & Nathan, 2012; Healy & Hoyles, 2000; Marghetis et al., 2014). Gestures produced during intellectual and social activity are a crucial form of multimodal discourse that reflect and impact a speaker's cognitive processes. Several researchers have shown that a learner's knowledge conveyed through gestures can be different from the knowledge conveyed through speech (Church & Goldin-Meadow, 1986; Goldin-Meadow et al., 1993), and that some knowledge is conveyed only through gestures (Hegarty et al., 2005; Pier et al., 2019).

In the following case studies, we investigated two conceptual gesture-speech mismatches (among many examples) from two college students who produced incorrect verbal insights for a geometric conjecture while simultaneously producing correct gestural insights. Specifically, we explored how these gestural insights (i.e., non-verbal expressions) complement ways students conceptualized and embodied geometric thinking.

Theoretical Framework

In pioneering work, Goldin-Meadow and colleagues found that children may express their understandings of mathematical concepts gesturally before they are able to articulate their understandings verbally. In fact, children often gesture correctly about emerging concepts while providing a verbal response that is incorrect or incomplete. For example, children learning to think algebraically may gesture their awareness of the need to balance two sides of an equation before they can verbally express the equality relation. These *mismatches* between gesture and speech are indicative of a transitionary period of knowledge acquisition and conceptualization, where gestures reveal the "leading edge" of students' conceptual learning (Church & Goldin-Meadow, 1986; Goldin-Meadow et al., 1993). Further, they reveal how learners' understanding of mathematical concepts is embodied.

Activation of sensorimotor neural systems can influence one's cognitive states (Thomas, 2013). Nathan (2017) refers to this process as *action-cognition transduction* (ACT) and identified two testable predictions: (1) *Directed actions*, physical movements that learners are directed to perform as part of an intervention, can induce cognitive states that prompt reasoning and problem-solving (Nathan & Walkington, 2017); and (2) directed actions can influence cognitive processes through nonverbal means. Leading up to the ACT hypothesis, Nathan et al. (2014) found that performing task-relevant arm movements prior to proof activities contributed to superior mathematical insight, illustrating how mathematical ideas are often expressed through non-verbal and verbal forms of reasoning.

We distinguish between the directed actions we may elicit from learners and the spontaneous gestures learners produce while communicating their thinking. Directed actions and spontaneous gestures can each engage body-based systems (typically hands and limbs) to represent mathematical objects and their transformations. Gestures are forms of bodily action (Alibali et al., 2014) that spontaneously co-occur with speech and thought (McNeill, 2008; Chu & Kita, 2009) and primarily effect one's verbal, deliberative forms of reasoning. Alternatively, directed actions are planned interventions that are not spontaneous and do not necessarily coordinate with learners' speech. They can, like gestures, affect learners' intuitions and insights

(Nathan & Walkington, 2017). As scaffolds, directed actions can even prime gesture production during a learner's explanations (Donovan et al., 2014). For example, Cook & Goldin-Meadow (2006) showed that either observing or copying a teacher's movements that represent an equalizer strategy for equivalence problems increased both the type and number of gestures that students spontaneously produced during instruction. Performing gestures, in turn, led to better problem-solving performance on a post-test, compared to students who expressed a solution strategy in speech only.

The current study documents cases of nonverbal, motor-based forms of understanding which we identify as "gestural insights." We explore three research questions: (RQ1) How do students express mathematical insights through their movements? (RQ2) How can directed actions contribute to mathematical insights? (RQ3) How can directed actions prime gestural insights?

Methods

Participants & Procedure

As part of a larger study investigating the role of directed action and prediction in geometric reasoning, undergraduate students (N = 127) were recruited from a large Midwestern university in the United States. Prior to arriving, participants were randomly assigned into one of four groups using a 2x2 between-subjects design: perform *directed actions* (DA=Yes; DA'=No) and generate *predictions* (P= Yes; P'=No): (1) Condition DA+P, participants mimicked an incomplete sequence of directed actions and then were asked to predict a "possible" third action; (2) DA+P', participants mimicked the series of three directed actions but made no predictions; (3) DA'+P, participants were not exposed to directed actions but were subsequently asked to "imagine" movements that could enact the geometric transformation of each conjecture; (4) in

DA’ +P’ (i.e., the control group), participants received no directed actions and made no predictions.

Participants took part individually in experimental sessions via the *Blackboard Collaborate Ultra* software platform. This study was run exclusively online because of the COVID-19 pandemic. Participation was video recorded. For each task, participants were prompted to read a geometric conjecture (see Figure 1). Directed actions for each conjecture were created using the pose editor, a resource from *The Hidden Village* (THV), an embodied video game to promote mathematical reasoning (Nathan & Swart, 2020; Figure 2). The set of directed actions for each conjecture was derived from prior observations of students' spontaneous gestures during successful proof production (Nathan & Walkington, 2017; Figure 3).

Participants completed each task by answering a prompt to consider the statement's veracity (i.e., always true or ever false) and provide their verbal justification. After completing all eight conjectures (see Table 1), participants were asked to complete a series of surveys about general geometry knowledge, spatial reasoning, interest in geometry, and demographic information.

Data Sources, Coding & Case Selection

Video recordings of participants’ online actions and verbal responses were transcribed and coded for correct mathematical insights expressed through movements and speech, based on pre-established criteria. *Gestural insight* had two defining characteristics: (1) mathematically relevant movements while a participant verbally justifies their evaluation of the truth of a geometric conjecture; and (2) the spoken justification in isolation does not invoke relevant, generalizable mathematical properties.

Across the corpus, we identified 42 cases of gestural insight for given conjectures, generated by 33 participants (see Table 2). For the current investigation, only participants from the DA + No P group who generated gestural insight were selected since these participants mimicked directed actions but were not prompted to predict the continuation of these movements. This selection allows us to focus on the possible effects of performing directed actions while excluding the possible effects of making a prediction.

The current study focuses on gestural insights for one conjecture — *The diagonals of a rectangle always have the same length*. This conjecture is neither the most nor least likely to elicit gestural insights (Table 2), and therefore provides a valuable illustration of the gestural insight phenomenon. From this subset, researchers identified two different ways that participants demonstrated their gestural insights: mimicking the directed actions from the intervention or producing spontaneous gestures that did not mimic the directed actions. We selected two cases that illustrate each of these two forms of gestural insight.

Gestures produced while proving each conjecture were coded as *representational* and *dynamic depictive gestures.* Representational gesture codes apply when participants depict semantic content, either literally or metaphorically, by virtue of handshape or motion (Alibali et al., 2001). *Dynamic depictive gesture* codes capture motion-based transformations of mathematical entities (Garcia & Infante, 2012). For example, formulating a parallelogram using two vertical arms is a representational gesture. A dynamic gesture is coded when the arms lean, skewing the sides of the parallelogram. Previous research has demonstrated that dynamic gestures are significantly correlated with the relative chance of producing correct mathematical insights (Nathan et al., 2021).

Results & Discussion

Case 1: Directed Action mimicry

In this first case (Figure 4), the student (S1) first asserts that the diagonals of a rectangle must be the same length, which reflects their intuition about the conjecture (Transcript #1, Line 1). S1 then holds their arms vertically on both sides of their body to visually represent the rectangle they are mentally constructing (Line 2), which mimics the second pose in the directed action sequence (Figure 3.2). As S1 says, "I rotate one to create a diagonal" (Line 3), they perform a dynamic depictive gesture, rotating their right arm to create a diagonal. The end position mimics the last pose from the directed action sequence (Figure 3.3). S1 moves back to the original position with both arms vertical before making a diagonal with their right hand and matching the first pose in the directed action sequence (Figure 3.1). They explain that the diagonals will "hit [their] arm in the same spot, regardless of which side it's on" (Line 4). S1's gestures reveal their understanding that diagonals intersect the bottom corners of the rectangle and must therefore have the same base angles in order to intersect the top corners of the rectangle.

S1 makes no mention of the mathematical properties of rectangles, such as their congruent and parallel opposite sides. S1 verbally references a specific rectangle in Line 2 rather than creating a generalized rationale for why this conjecture must always be true. Thus, it is not apparent through this verbal explanation alone whether S1 understands why this conjecture is true. Non-verbally, S1 gestures that a rectangle has a set of congruent and parallel opposite sides (Line 2). S1 then shows that both diagonals must always be equal (Lines 3-4).

Case 2: Spontaneous Gestures

The student (S2; Figure 5) begins their verbal explanation by asserting the conjecture is true (Transcript #2; Lines 1-5). S2 then describes how the diagonals will each be the same length

(Lines 6-9). During this, S2 performs multiple representational gestures depicting the two diagonals using their right index finger to trace the diagonals in the air. Acknowledging their inability to verbally explain their reasoning (Line 7), S2 clarifies their thinking (Line 8). Again, S2 draws a diagonal with their right index figure (a third representational gesture), then uses both hands to "pick it up and turn it," forming the second diagonal.

Like S1, S2 does not provide a verbal explanation for why they believe the conjecture must be true, but their gestures reveal their mathematical reasoning. However, the cases of S2 and S1 differ in several ways. First, neither S2's verbal rationale nor gestures explore the properties of rectangles or similar triangles. S2 does not explicitly describe or represent this rectangle in gesture. While S1 and S2 both describe and demonstrate a rotation transformation, S2's gestures suggest an understanding of the rotational symmetry of a rectangle since S2's dynamic depictive gestures of rotating (or "turning") the diagonal (Line 8), produce an equivalent diagonal in the other corners. Second, S2 explicitly shows metacognitive thinking (Line 7), awareness that they do not have the ability to verbalize their understanding. This awareness may drive S2's rephrasing and production of different gestures (Lines 8-9). Third, S2 does not mimic the gestures from the directed action sequence. Instead, S2 uses their index finger to draw representations and uses both hands to (virtually) "pick up" and "rotate" the imagined diagonals using a dynamic depictive gesture. This gesture sequence suggests S2 did not recognize the relevance of the directed actions to the conjecture or felt that the directed actions were insufficient to demonstrate their understanding.

Significance

These cases show that students who were unable to provide verbal mathematical insights were able to demonstrate their understanding through gestures. Many studies on geometry proof processes—and mathematics more generally—focus on verbal descriptions of mathematical insights (e.g., Healy & Hoyles, 2000; Walkoe, 2019). We believe that a potentially rich source of nonverbal formative assessment information is often overlooked. Future work needs to explore the conditions under which gestural insights are produced and how they can inform teachers' instructional practices.

Gestural insights, often exhibited as gesture-speech mismatches, may indicate that students are starting to extend their current zone of proximal development (Goldin-Meadow et al., 1993). One case presented here demonstrates how directed actions may scaffold students' geometric reasoning, offering an embodied intervention for fostering conceptual reasoning. Future work will investigate if the findings presented here generalize with a larger sample size, the efficacy of direct actions as a scaffold for geometric reasoning, and the possible role of prediction in gestural insight and geometric reasoning.

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Tables & Figures

Table 1

Eight Geometric Conjectures

Labels	Conjecture Text	Verity
Triangle Inequality	The sum of the lengths of two sides of a triangle is always greater than the length of the third side.	True
Similar Triangle	For a triangle that is similar to triangle ABC, the side opposite to angle B must have the same length.	False
Parallelogram	The area of a parallelogram is the same as the area of a rectangle with the same length and height.	True
Diagonals	The diagonals of a rectangle always have the same length	True
Opposite Angles	The opposite angles of two lines that intersect each other are always the same	True
Angles Sides	In triangle ABC, if Angle A is larger than Angle B, then the side opposite Angle A is longer than the side opposite Angle B	True
Halve Rectangle	If you double the length and width of a rectangle, then the area is exactly doubled.	False
Rotate Reflect	Reflecting any point over the x-axis is the same as rotating the point 90 degrees clockwise about the origin.	False

Table 2

Cases of Gestural Insight per Conjecture Across the Four Experimental Conditions

Conjecture	DA + P	**DA + No P**	No DA + P	No DA + No P	Total
Triangle Inequality	0	**0**	0	1	1
Similar Triangle	0	**0**	0	0	0
Parallelogram	1	**0**	1	0	2
Diagonals	2	**5**	1	1	9
Opposite Angles	6	**6**	9	3	24
Angles Sides	1	**2**	3	0	6
Halve Rectangle	0	**0**	0	0	0
Rotate Reflect	0	**0**	0	1	1
Total	10	**13**	14	5	42

Note. DA = Directed Action condition. P = Prediction Condition. Only participants from the DA + No P group were examined in this investigation.

Figure 1

A Geometric Conjecture "The Diagonals of a rectangle always have the same length."

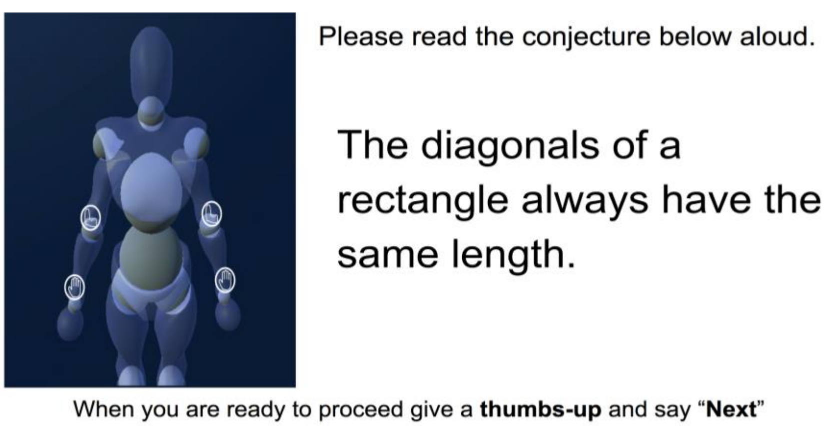
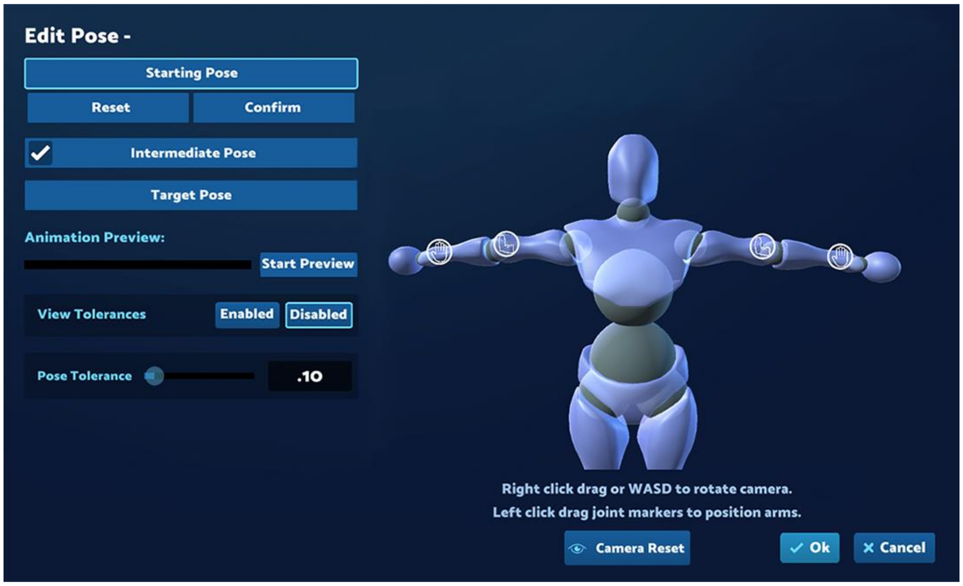


Figure 2

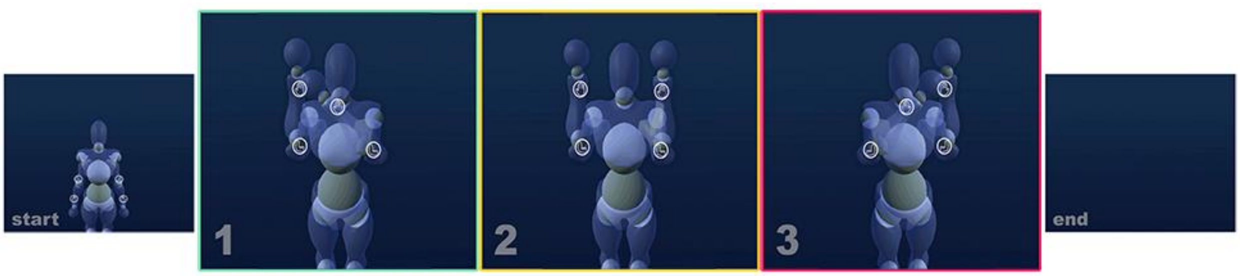
The Hidden Village Pose Editor



Note. The Hidden Village Pose Editor. *Center*. The user can manipulate the avatar into three poses (i.e., starting, intermediate, and target poses) to represent geometric objects and transformations relevant to the conjecture.

Figure 3

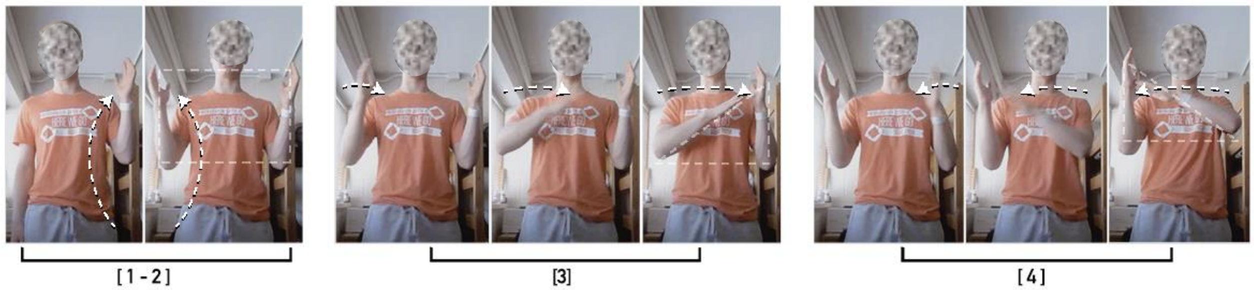
Directed Actions for the Diagonals conjecture



Note. The conjecture text reads, *"The diagonals of a rectangle always have the same length."* These directed actions are intended to convey a key insight related to this conjecture – that the diagonals must be congruent because they each are the hypotenuses of left and right triangles with equal arm lengths.

Figure 4

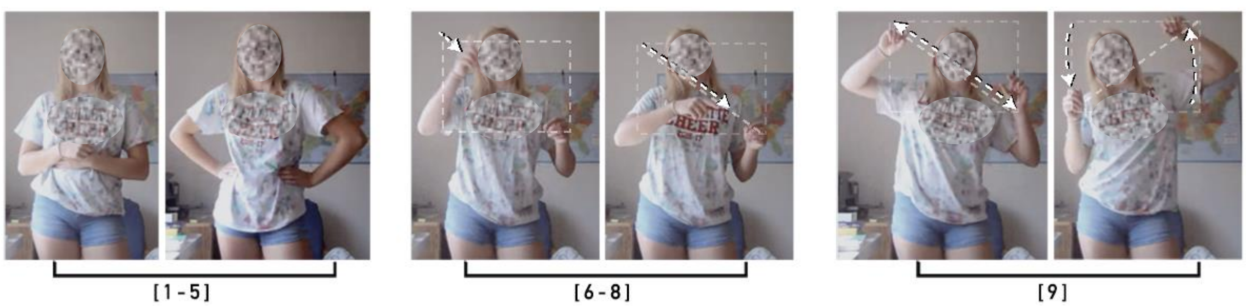
Transcript #1:



Transcript #1: [1] The statement is always true
[2] because when I hold a, make a rectangle of my arms,
[3] I rotate one to create a diagonal,
[4] it's gonna hit my arm in the same spot, regardless of which side it's on.

Figure 5

Transcript #2:



Transcript #2: [1] That seems to be true again.
[2] Oh my goodness, I haven't taken geometry in so long, but I think that.
[3] Now, I'm just picturing the rectangles in my head.
[4] I'm gonna say true.
[5] And I'm just, it's just a rectangle, it's a rectangle.
[6] so if you go from either the diagonals, this is gonna, it's gonna be the same.
[7] I don't really know how to explain it very well,
[8] but all the rectangles that I can picture in my head, the diagonal, both the diagonals would be the same.
[9] Like if you took one diagonal and you just like picked it up and turned it and put it on the other diagonal, that would be the same.