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Directed Actions Scaffold Gestural Insights in Geometric Reasoning

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Abstract: Spontaneous gestures often express thoughts that differ from speech. Priming students to perform *directed actions*, movements designed to foster conceptual understanding, may give a way for students to organize their thoughts by providing a gestural vocabulary. G*estural insights* are nonverbal expressions of movement that reveal students' mathematical reasoning even when they were unable to verbalize their insights. This case study demonstrates how body-based resources like directed actions can foster gestural insights that support conceptual thinking.

Introduction and Theoretical Background

During mathematical reasoning, teachers and students naturally communicate key ideas using descriptive language, verbal inference, and gestures of the arms and hands (Alibali & Nathan, 2012). These co-speech gestures are a crucial form of multimodal discourse that reflect cognitive processes. Researchers have shown that the knowledge conveyed through gestures can differ from and complement speech (Goldin-Meadow et al., 1993; Pier et al., 2019). These *gestural insights* are phenomena in which students' gestures exhibit mathematical insights that are otherwise absent in their speech. Specifically, this case study explores how embodied thinking in the form of gestural insights contributed to a student's mathematical conceptualizations while generating geometric proofs.

Spontaneous co-speech and co-thought gestures often express one's cognitive processing (McNeill, 1992). Nathan et al. (2014) found that performing cognitively relevant arm movements contributed to superior insights into mathematical proofs. Thus, scholars hypothesize that directing learners to perform movements may provide effective embodied interventions that improve reasoning and learning (Nathan & Walkington, 2017). *Action-cognition transduction* proposes that instructing learners to perform *directed actions* will influence one's reasoning in ways cognitively congruent with the actions performed through nonverbal means (Nathan, 2017). Therefore, we ask: How do directed actions contribute to gestural insights and mathematical understanding?

Methods

This case comes from a larger study investigating the impact of directed actions and predictions on mathematical reasoning and focuses on one conjecture — *The diagonals of a rectangle always have the same length* (Figure 1; Xia et al., 2022a). The participant was prompted to first perform directed actions and then read the geometric conjecture, consider its veracity, and provide a verbal justification. Recordings of participants' verbal responses and actions were transcribed and coded for correct mathematical reasoning, gestural insights, representational gestures, and dynamic depictive gestures (Xia et al., 2022b).

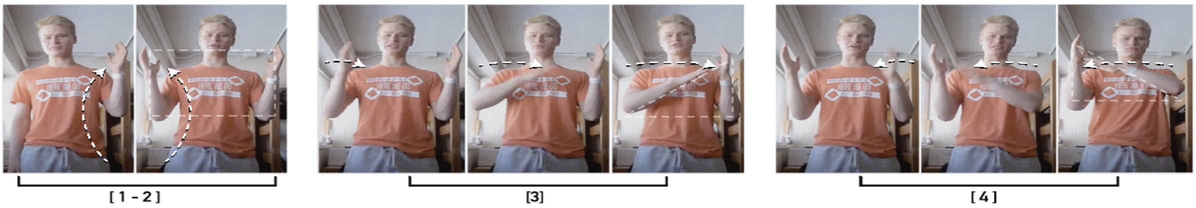
Figure 1
Directed Actions for the "The diagonals of a rectangle always have the same length." conjecture

Note. These directed actions are intended to convey a key insight related to this conjecture – that the diagonals must be congruent because they each are the hypotenuses of left and right triangles with equal arm lengths.

Results

Student 1's (S1; Figure 2) intuition reflects their assertion that the diagonals of a rectangle must be the same length (Line 1). Mimicking the second pose from the directed action sequence (Figure 1, Frame 2), S1 arms represent their mental construction of the rectangle (Line 2). Next, they rotate their right arm to create a diagonal while saying "I rotate one to create a diagonal" (Line 3). S1's gestures reveal their understanding that diagonals intersect the bottom corners of the rectangle and must therefore have the same base angles intersect the top corners of the rectangle (Line 4). S1 does not provide a generalized rationale for why this conjecture must always be true as it makes no mention of the mathematical properties of rectangles. Non-verbally, S1's gestures demonstrated that a rectangle has a set of congruent and parallel opposite sides (Line 2), expressing a generalized understanding that both diagonals are equal (Lines 3-4).

Figure 2
Transcript #1



[1] The statement is always true [2] because when I hold a, make a rectangle of my arms, [3] I rotate one to create a diagonal, [4] it's gonna hit my arm in the same spot, regardless of which side it's on.

Discussion & Conclusions

This case shows that students who were unable to provide verbal mathematical reasoning demonstrated their emerging understandings through gestures. With these findings in mind, we revisit the research question.

This case shows how mathematical understanding is conveyed through movement. S1's geometric construction depicts two equivalent diagonals originating from consecutive corners of the rectangle. This insight highlights how dynamic gestures expand students' ability to convey their mathematical understandings. S2 integrated the directed action sequence in their gestures in their mathematical reasoning. This integration suggests S2 may have recognized the cognitive relevance of the directed actions to the conjecture. Moreover, S2 expressed generalized features in their gestures that were not found in their speech. In this case, we see how directed actions scaffolded the student's geometric reasoning and revealed potential for embodied interventions to help elicit gestural insights that foster conceptual reasoning.

Gestural insights may demonstrate ways students are extending their zones of proximal development (Goldin-Meadow & Church, 1993). Studies on geometry proof processes—and mathematics more generally—focus on verbal descriptions of mathematical insights (e.g., Xia et al., 2022). Students' gestures reveal nonverbal ways of knowing that could be used to guide learner-centered instruction by expanding formative assessment practices to include embodied expressions of knowing.

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