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The Role of Action-Prediction in Mathematical Reasoning

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Abstract: Action-cognition transduction (ACT) posits how body movements support actors' inference making by activating feedforward and feedback mechanisms in consideration of plausible outcomes. Following ACT, we investigated whether prompting people to predict future body states (i.e., feedforward processes) facilitates mathematical reasoning. In this randomized 2x2 factorial experiment, undergraduate students (N = 127) *performed directed actions* (Yes/No) or *generated predictions* (Yes/No) prior to justifying the veracity of geometry conjectures. As predicted by ACT, directing participants to mimic cognitively relevant directed actions facilitated mathematical insight, though moderated by task demands. Moreover, merely prompting participants to predict movements, with or without performing directed actions, also enhanced proof performance. This study provides insights into how body-based simulation induces inference-making in support of conceptual understanding for mathematical transfer and generalization.

Introduction

Theoires of grounded and embodied cognition (GEC, Barsalou, 2008) adhere to the premise that cognitive processes emanate from and are influenced by one's body-based actions and simulations of actions. Goal-directed actions engage both feedforward and feedback processes that assess current states of the world in order to anticipate all of the plausible future actions, thereby integrating information from both actual and anticipated outcomes (Pezzulo, 2008). The continuous interplay between acting in the world and anticipating consequences of those actions illustrates the close reciprocity between sensorimotor and cognitive processes (Neisser, 1976).

In the current experiment, students observed the movements performed by animated avatars and were prompted to predict which body movements the avatars will make next. It is hypothesized that engaging in these action-predictions activates spatial-motoric processes that facilitate one's reasoning about space and shape (Nathan & Walkington, 2017). By connecting anticipatory nature of actions with reasoning processes, this study investigated whether action-prediction influences people's mathematical reasoning in geometry proof.

Theoretical framework

One of the core assumptions for embodied learning is that cognitive processes operate as a *predictive architecture*. People do not passively wait for input to act but continually anticipate what is to come in streams of sensory input, and we are poised to respond (Clark, 2015). *Action-Cognition Transduction* (ACT; Nathan, 2017) is consistent with this philosophical view of cognition, by framing action and cognition as reciprocal processes operating in close concert: Thoughts drive goal-directed actions; and actions induce cognitive states commensurate with those goal-directed actions. ACT offers an account of how actions and simulations of actions activate feedforward (i.e., predictors) and feedback (i.e., reaction) mechanisms that anticipate and respond to outcomes, thereby influencing the actor's behaviors and cognitive states.

When the actions are deemed as *mathematically relevant* they can contribute to enhanced mathematical reasoning. This occurs in cases of spontaneous co-speech and co-thought gestures (Nathan et al., 2021). It also occurs in cases when people are prompted to perform specific mathematically relevant *directed actions* as part of a designed intervention (Nathan et al., 2014). Some studies have shown that directed actions can prime gesture production during learners' explanations, which, in turn, contribute to superior cognitive performance (Donovan et al., 2014). Similarly, Nathan and colleagues (2014) found that compared to those who performed unrelated arm movements, participants who were directed to perform conceptually related movements were more likely to generate key mathematical insight for conjectures. However, similar to Walkington et al. (under review), relevant directed actions by themselves may not directly improve mathematical reasoning, but do so through moderated effects of "gestural replays" of these actions during their multimodal explanations.

Prior results suggest that relevant directed actions can prompt reasoning through nonverbal means. However, their connection to more complex processes like proof production might require additional pedagogical tools to fulfill their purpose. Prediction, or the combination of directed actions and prediction, might be needed to support students' reasonings about mathematical transformations. Thus, we investigated three research questions: (RQ1) Does predicting actions impact students' geometry insight, proof, or gesture?

(RQ2) Does performing relevant directed actions impact students' geometry insight, proof, or gesture? (RQ3) Does predicting relevant directed actions that they perform impact students' insight, proof, or gestures?

Methods

Participants

Participants included adult students (*N* = 127; 63.0 % female) recruited from a large university in the Midwestern United States. Out of these participants, 33.9% had not taken Calculus I, 46.5% had completed or were enrolled in Calculus I or II, and 19.6% had taken or were taking a mathematics class above Calculus II.

Procedures

Participants were randomly assigned into one of four groups using a 2x2 between-subjects design: (1) perform *directed actions* (DA=Yes; DA'=No) and (2) generate *predictions* (P=Yes; P'=No). They took part individually with an interviewer. After going through the instructions and completing a practice-trial example conjecture, participants started the actual conjecture tasks. Participants were prompted to read a mathematical conjecture (see an example in Figure 1). Next, they adhered to one of four conditions: (1) Condition DA+P' participants (n=30) mimicked the complete series of three directed actions without any prompt to make predictions (see directed actions in Figure 1); (2) DA+P participants (n=30) mimicked an incomplete sequence of directed actions and were then shown the "?" symbol and asked to predict a "possible" third movement; (3) DA'+P participants (n=37) were not exposed to any directed actions but were asked to "imagine" movements that could enact the geometric transformation of each conjecture; (4) DA' +P' (i.e., the control group; n=30) participants received no directed actions and were not prompted to make predictions. Finally, participants completed each conjecture by answering a prompt to consider the statement's veracity (i.e., *always true* or *false*) and to provide a verbal justification. Each participant completed a set of eight conjectures with order counter-balanced.

After giving video-recorded responses to the eight conjectures, each participant was asked to complete surveys about demographics, general geometry knowledge, and spatial reasoning. Prior study have revealed that gender, math expertise, and spatial ability are correlated with geometric reasoning (Nathan et al., 2021).

Figure 1

An example conjecture "If you halve the length and the width of a rectangle, then the area is exactly halved."

Note. These directed actions are intended to convey a key insight related to this conjecture – that the area would actually be smaller than a half by halving both the length and width.

Coding

Video recordings of participants' responses were transcribed and coded. Insight (reliability κ = .93) was measured by the participant's understanding of key mathematical ideas for each conjecture. Each participant's verbalized proof (reliability κ = .96) was coded following Harel and Sowder's (2005) three criteria: (1) *generality* of the argument, (2) use of *operational thinking*, and (3) exhibit a chain of *logical inference*.

Representational gestures (Alibali et al., 2001) were classified as either non-dynamic or dynamic depictive gestures (reliability κ = .95). *Non-dynamic gestures* reflect only static properties of the mathematical entities, such as tracing along a shape. *Dynamic depictive gestures* enact motion-based transformations of mathematical entities, such as dilating triangles (Garcia & Infante, 2012).

Results

Data analysis employed mixed-effects logistic regression models for binary outcomes on insight, proof, non-dynamic and dynamic depictive gestures. Models were fit using the *glmer* command of the *lme4* package in R. *Participant ID* and *Conjecture* were included as random effects. All models included the *experimental condition* as the primary predictor. Three student characteristics, *gender*, *students' most advanced previous math course*, and *spatial score,* were retained in the models as covariates that significantly improved the fit of our models. Finally, we report odds ratios that are exponentiated raw coefficients.

Research Question 1: Effects of prediction on performance

To address RQ1, we used the experimental condition prediction (P) vs. no prediction (P') as the main predictor. The regression model predicting insight showed that having a math course above Calculus II strongly predicted insight (OR = 4.48, d = .82, p <.001). Similary, having a math course above Calculus II (OR = 1.59, d = .25, p = .048) and high spatial scores (OR = 1.17, d = .09, p = .001) were associated with valid proofs. Consistent with the primary hypothesis of this investigation, students who predicted the movements, whether or not they actually performed them, were significantly more likely to formulate a mathematically valid proof (OR = 1.45, d = .21, p = .035), even when controlling for gender, prior math courses, and spatial ability.

Additionally, the models predicting gestures show that the spatial score was positively associated with dynamic depictive gestures (OR = 1.17, d = .08, p = .007) while negatively associated with non-dynamic gesture (OR = .9, d = -.05, p = .018). However, the prediction condition (with prediction versus no prediction) was not a significant factor for predicting the production of dynamic depictive gestures or non-dynamic gestures.

Research Question 2: Effects of directed actions on performance

To address RQ2, we used the experimental condition of performing cognitively relevant directed actions (DA) vs. no directed actions (DA') as the main predictor. As with the model used for RQ1, having a math course above Calculus II predicted insight (OR = 4.56, d = .83, p < .001). And the spatial score still predicted proof performance (OR = 1.16, d = .08, p = . 002). The models predicting gestures showed that males were less likely to produce dynamic depictive gestures (OR = .63, d = -.25, p = .045). Spatial scores predicted dynamic depictive gestures (OR = 1.17, d = .09, p = .006), while negatively associated with non-dynamic gesture (OR = .90, d = -.05, p = .017). However, the experimental condition (DA vs. DA') was not a significant predictor of insight, proof, or gestures. The results, which were not collected during game play, are not consistent with studies of embodied video game play showing advantages of directed actions (Nathan & Walkington, 2017).

Considering the overall high performance on geometric reasoning (i.e., around 70% produced correct insight), we further analyzed participants' behaviors (n = 381) on three conjectures that showed a relatively lower percentage of correct proof performance (less than 33%) but did not show a floor effect (higher than 10%). The results showed that performing directed actions predicted insight (OR = 1.85, d = .34, p = .043) for these difficult conjectures. However, there were no differences in proof performance or gesture production with or without relevant directed actions. These results show that participants performing relevant directed action were more likely to generate insight compared to those who did not while justifying difficult conjectures.

Research Question 3: Effects of predictions of directed actions on performance

To address RQ3, we used four experimental conditions as the main predictor. Having a math course above Calculus II was still a strong predictor of insight (OR = 4.45, d = .83, p < .001). When controlling for prior math courses, there was a trend that participants were more likely to produce correct insight if they predicted the relevant directed actions that they performed (DA+P; OR = 1.65, d = .28, p = .081). For proof, males and high spatial scores predicted proof performance (OR = 1.61, d = .26, p = .045; OR = 1.16, d = .09, p = .002, respectively). Similarly, there was a marginal trend suggesting that participants were more likely to construct a valid proof if they predicted movements without directed actions (DA'+P; OR = 1.54, d = .24, p = .080).

Gesture models show that males were less likely to produce dynamic depictive gestures (OR = .63, d = -.25, p = .045). High spatial score was associated with dynamic depictive gestures (OR = 1.17, d = .09, p = .006) but negatively associated with non-dynamic gestures (OR = .91, d = -.05, p = .021). However, the experimental condition was not a significant predictor of either dynamic or non-dynamic gestures.

Discussion

One of the central premises of embodied theories of learning is that people continuously operate in an anticipatory manner (Clark, 2015). This predictive stance allows us to engage in simulated actions that model how the state of the world—us included—will change in response to our behaviors. Because our motor system is constantly projecting how our actions will change the world, Action-Cognition Transduction (Nathan, 2017) is one account for how actions can construct grounded, situated "abstr-actions" (Abrahamson et al., 2020).

In answer to RQ1, as predicted by ACT theory, prompting people to predict actions led to significant advantages for formulating valid mathematical generalizations. In response to RQ2, mimicking cognitively relevant directed actions helped students generate accurate mathematical insight about difficult conjectures. The benefits for difficult tasks suggest this is where cognitive processes may benefit most from both actions and simulating actions. Concerning RQ3, predicting cognitively relevant actions that were also performed showed only marginally significant benefits above and beyond predicting those actions. These findings suggest that action-prediction may facilitate participants' mathematical reasoning by simulating the future states of geometric transformations needed to support mathematically valid generalizations and proofs.

These findings contribute to the emerging theories of grounded and embodied cognition (GEC) in two ways. First, our results provide direct evidence to support ACT theory: motor behaviors and cognitive processes are closely coupled; movements – even when they are simply imagined – can benefit cognition. Second, this study extends prior research by providing an interactive environment where students engaged in actions and action-predictions as a path toward more advanced reasoning. Prior research has shown that students' proof performance was improved when they were told that the directed actions were mathematically relevant (Nathan et al., 2014). Because action-prediction is an integral component of most action sequences, it is rare to methodologically isolate the influences of action-prediction from action on thinking, as was done here.

This study has several limitations. One is that there may be additional interpretations of these results other than ACT, as a rich set of cognitive and post-cognitive frameworks for embodied learning emerge (e.g., Abrahamson et al., 2020; Danish et al., 2020). Another is to explore replication of these findings with a broader set of tasks and participants, and in face-to-face settings.

Nonetheless, these findings offer implications for mathematics education. First, there is educational value in engaging learners in mathematically relevant actions. Second, math educators can capitalize far more on the value of imagination and action-prediction for fostering reasoning that supports inference making. Third, the emerging designs of embodied interventions offer students active contexts and invite students to step into their roles as interactive learners who utilize rich body-based resources for supporting mathematical transfer.

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