

Assessing Engineering Students' Embodied Knowledge of Torsional Loading Through Gesture

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Abstract:

This full paper concerns the use of gesture analysis to guide instructional approaches in engineering education. Engineering is rife with abstract mathematics and processes for quantifying physical phenomena. In engineering instruction, *formalisms first* is a practice that privileges formalisms over grounded and applied ways of knowing that are common in engineering curricula. By way of contrast, *progressive formalization* is an alternative pedagogical practice that intentionally grounds the meaning of mathematical formalisms in one's sensorimotor experiences in order that the formalisms are meaningful to learners. In the courses of explaining engineering concepts, instructors often make iconic gestures (gestures that represent objects, actions, and relationships) that are based in perception and action as a means for grounding domain knowledge prior to introducing formalisms. In response, students' gestures can be either concordant (i.e., conceptually aligned) or discordant (i.e., conceptually misaligned). The latter, also known as gesture-speech mismatches, are indices for states of transitional knowledge in which learners exhibit a readiness to learn. Thus, the current research observes the spontaneous gestures students make while describing torsional loading and investigates the added benefits of incorporating gesture into formative assessments of engineering education. Results indicate that students do use gestures as integral parts of their explanations in an engineering lab setting and that gestures and co-articulated speech were often matched. Instances of gesture-speech mismatches provides instructors opportunities to assess student knowledge, knowledge-in-transition, and initial learning and correct understandings prior to summative assessments.

Introduction:

Engineering knowledge is often steeped in mathematical and scientific formalisms (i.e., equations, symbols, diagrams, technical language, etc.). In actuality, these formalisms constitute the conventions that engineers use to communicate the knowledgebase that stem directly from the physical phenomena they work to manipulate. Consequently, engineering education centers around a pedagogy that focuses first on the formalisms that abstract real phenomena. Rather than building on an understanding of the phenomena from their direct, primary experiences, engineering students often are reasoning in terms of *formal redescriptions* of these phenomena [1]. This *formalisms first* approach [2], we contest, exposes students to ideas in terms of their

formalisms before students even get opportunities to ground [3] their intuitions in the experiences that underlie conceptual understandings [4-6].

Theoretical Framework:

Analyses of secondary-level math textbooks show that they overwhelmingly introduce concepts in equation form, requiring students to exhibit mastery by manipulating symbols long before presenting the same ideas in contexts of real-world problems [7, 8]. This pedagogical pattern of formalisms first persists despite data showing that students (even very high achievers) perform far better on tasks that concretize presentations in lieu of carefully matched symbolic formalisms [9, 10]. Fortunately, new approaches towards engineering education propose a new pedagogy of *progressive formalization* [2] to counter the formalisms first practice.

Progressive formalization provides experiences that intentionally ground the meaning of mathematical formalisms in one's sensorimotor experiences to make the formalisms meaningful to learners. For example, a progressive formalization approach may introduce students to objects arranged on a physical pan balance and show that removing identical objects simultaneously from both sides maintains equilibrium (see Figure 1). This physical phenomenon can then be depicted iconically, with images of objects "balanced" on a diagrammatic pan balance, symbolically as equations using letters and arithmetic operators that *formalize* the general notion of maintaining an equal relationship through symbol (and object) manipulation [11], or even gesturally using body-based movements. In fact, gestures are a powerful communicative resource for grounding mathematical and scientific principles. In this example, a student may use their hands to enact the equilibrium exhibited by the pan balance apparatus. This type of *simulated action* [12] can reveal the sensorimotor basis of students' reasoning processes as they engage in analysis and problem solving [13].

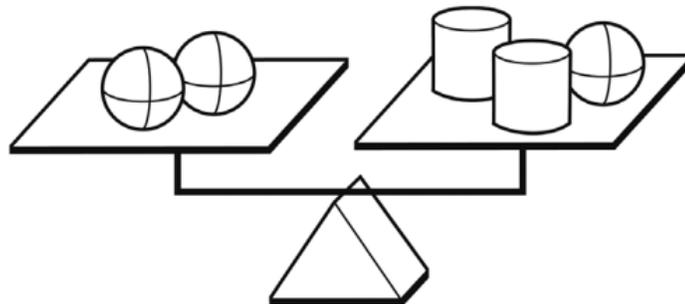


Figure 1: Pan balance in a state of equilibrium. Spheres represent variable 'S' while the cylinders represent variable 'C' in the formal equation: $2S = 2C + S$ [10].

To back up momentarily, gestures are defined in the psychology literature as spontaneous, co-speech movements of the hands and arms that convey a person's knowledge state [13, 14]. There is ample evidence that gestures can facilitate learning and teaching in STEM [15-17], and specifically in engineering [18]. Engineering students, as well as their instructors, frequently produce gestures while reasoning about physical and mathematical phenomena [18]. Gestures that occur during engineering activities often convey some of engineers' practical knowledge in

nonverbal forms as complements that enrich their verbal description and or symbolically represented ways of knowing and communicating.

As a separate modality from speech, gestures' relationship to speech can be either *concordant*—redundant in the meaning they each express, or *discordant*—offering complementary information about one's state of knowledge. In the learning sciences, careful analyses has shown that instances of gesture and speech that are discordant often reveal states when a person is literally of “two minds” and thinking of a phenomenon in two different ways—one revealed through speech and the other through gesture. Such discordant events are important because researchers have shown that they index a learner's “transitional state of knowledge” when someone is just on the edge of learning a new concept and reconceptualizing their old ways of thinking [19]. Discordant gestures occur when verbal and nonverbal processes are operating differently; this can occur when: (1) verbal explanations correctly convey concepts, but gestures do not convey the same information, (2) when accurate gestures are misaligned with incorrect verbal explanations, or (3) when they are both correct or incorrect, but for different reasons.

For example, Church & Goldin-Meadow [20] observed children who were just developing their understanding of conservation. When liquid was poured from a tall-thin vessel into a wide-short vessel, some children expressed verbally that there was “less” liquid in the shorter vessel while simultaneously gesturing with their hands that the shorter vessel was wider (presumably expressing they are aware of the change in dimensions). Children who made this discordant gesture (i.e., saying “less” while gesturing “wider”) were statistically more likely than students whose incorrect speech was concordant with their gestures. In this way, being of two minds is indicative of a cognitive transition when a person is ready to learn [21]. Thus, students can simulate the grounded foundations of abstract concepts like the law of conservation that they may not otherwise be able to easily explain verbally.

In engineering education, assessments often privilege learners' ability to express their knowledge through language and symbols. These traditional assessment practices neglect important information about the dynamics of students' knowledge when they are in transition. Implementing formative assessment practices that explicitly attend to students' discordant gestures during engineering activities have the potential to greatly improve instructional effectiveness and student learning.

From this, we posit that students' spontaneous gestures can provide engineering educators insights into the verbal and non-verbal processing of learners conceptual understandings during the activities in a Mechanics of Materials lab course. We used this framing to investigate the following research questions (RQ): (1) Do engineering students use spontaneous gestures to convey ideas? (2) Is there added value by documenting students' concordant and discordant gesture-speech for formative knowledge assessment? Moreover, we hypothesize: (1) Students will produce gestures—whether concordant or discordant—that convey their ideas, and these gestures will reflect their current understandings of torsion; (2) Some of the students' knowledge is encoded in nonverbal forms (i.e., gestures) and thus incorporating gesture information is expected to provide a richer and more complete assessments of students' emerging knowledge, related to torsional concepts, that be used to guide instruction.

Methods:

Participants. Engineering students (N=4) who previously passed or who were concurrently enrolled a mechanics of materials course from various disciplines (mechanical, engineering physics, and civil and environmental engineering), grade levels (sophomores to seniors), and coursework experience were recruited to participate in a pilot study and separated into two groups each consisting of two students.

Procedure. Students completed a video-recorded pre-lab assessment on torsion during which the following questions were asked:

1. You are curious to know which material will respond better under a torsional load. The samples are consistent in shape and size; only the material changes. The program for the test permits consistent angular displacements regardless of the material being tested. What ways can you determine how the material responds to torsional loading? Describe any indicators in the experiment that can provide relevant information.
2. How was energy added to the specimen in the lab? How was energy released from the specimen? Describe these processes as clearly as you can.
3. Describe where the maximum shear stress occurs on the sample due to torsional loading. Why does maximum shear stress occur at this location? What information does this provide about the response of the material under torsional loading?

After answering the questions, students were prompted by the lab instructor to explain their responses, which were also recorded. Following the pre-assessment, students completed a lab activity on torsional testing. The lab activity consisted of each group testing samples of metallic rods (aka, a dog-bone sample) using an ADMET material testing system (Figure 2.a) with a straight line drawn across the gauge length of the undeformed A36 steel specimen—a ductile material (Figure 2.b); material testing concluded at failure (Figure 2.c).

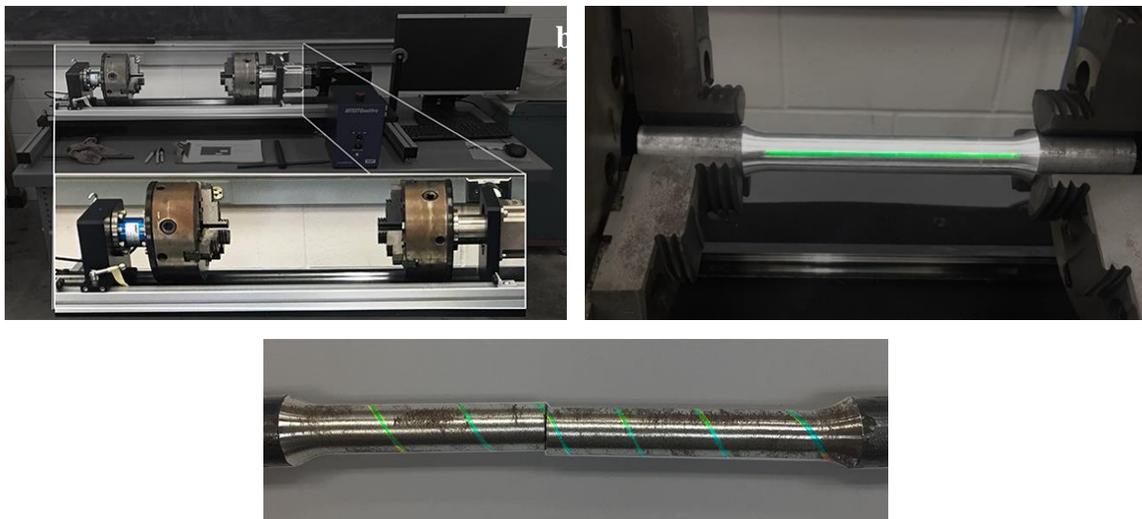


Figure 2: Equipment and testing results of torsional loading. (a) Displays the ADMET material testing system. (b) shows an undeformed sample; the green line provides students a visual representation prior to deformation. (c) an image of a deformed sample with the green line providing students a visual representation of deformation at fracture.

Data Collection. Video recordings of students' speech and gesture were transcribed, coded, and then analyzed qualitatively using grounded theory [22]. Accuracy of students' mechanical explanations was documented in two ways: (1) based on speech-alone; (2) based on speech and contemporaneous co-speech gestures. In addition, gestures were coded as being either concordant (i.e., conceptually aligned) or discordant (i.e., conceptually misaligned) with speech, with particular attention given to tracking discordant gestures as evidence of knowledge in transition [19].

Results:

The first research question (RQ1) investigates whether students produced spontaneous gestures during their explanations, and, if so, how these gestures expressed their reasoning? We hypothesized that students would produce gestures while discussing their conceptual understanding of torsion during the pre-lab and instructor-guided discussion. To investigate this first research question, verbal processing and gestural data were collected from the video-recorded pre-assessment and the preceding discussion with the lab instructor. Analysis of the video recordings indicate that students use a multitude of gestures during their explanations with classmates. Table 1 on the next page summarizes the gesture codes, a description for each, their frequencies, and the number of instances of discordant speech and gesture. The most common coded gesture was *rotational (loading)*, movements of the hands and arms that simulate the angular displacement of a sample loaded in torsion, and other higher frequency gestures used included *geometric shape*, *rotational (unloading)*, *increase*, *progression*, *magnitude (increase)*, and *cross-sectional cut*. Gesture with low frequency of use were coded during analysis such as *fracture*, *parallel*, *decrease*, *angle*, *release*, *clamp*, *shearing*, and *magnitude (decrease)*; these are worth noting but not highlighting.

Table 1: Common co-speech gestures used to depict conceptualizations of torisonal loading.

Gesture Code	Description	Frequency of Use	Instances of Discordancy
Rotational (Loading)	Rotating hands and or arms <i>away from</i> body to describe angular displacement	34	5
Geometric Shape	Indicating shape (2D, 3D, or graphical representations) of object or concept being described	28	1
Rotational (Unloading)	Rotating hands and or arms <i>towards</i> the body to describe angular displacement	22	2
Increase	Used to show a one-off increase of idea (e.g., numerical) with movement <i>away from</i> the body	14	0
Progression	Procedural progression of ideas or actions within a concept	9	0
Magnitude (Increase)	Progressive increase in magnitude of hand movement	5	0
Cross-Sectional Cut	Used when discussing torque on cross-sectional areas	5	0

The second research question (RQ2) investigated whether documenting students' concordant and discordant gesture-speech alignment added value as formative assessment of students' emerging knowledge. We hypothesized that some of students' knowledge is encoded in nonverbal processes, such as gesture, and through incorporation of gesture information upon assessment it is expected that a richer picture of students' emerging knowledge is developed. Furthermore, the addition of gesture information can be used to guide instruction. To investigate RQ2, speech and co-speech gestures were coded as concordant (alignment with conveyed ideas) or discordant (misaligned with conveyed ideas). Table 1 displays instances of discordant gestures. *Rotational (loading)* had the highest frequency of use and exhibited the most instances of discordancy. Other notable discordant gestures include *geometrical shape* and *rotational (unloading)*.

Figure 3 depicts a student explaining how energy is added to the sample rod during torsional loading. In this example, the student first uses a *geometrical gesture* to simulate a sample being tested. The student uses abstracted geometry to convey angular displacements through a series of concordant speech-gesture pairings, starting with the *geometrical shape* and *rotational (loading)* gestures while verbalizing “Well if the material’s [Geometrical Shape gesture, figure 3.a] in its elastic state, then if you twist it [Rotational (Loading) gesture, figure 3.b-d] and then...”, followed by *release* and *rotational (unloading)* gestures and “... you release [Release gesture, figure 3.e] it, it’s going to twist [Rotational (Unloading) gesture, figure 3.f] back really fast.” Finally, the student concludes, “That’s a release of energy right there.” Correctly, this student explained in speech and gesture how energy is added through angular displacement and a possible energy release mechanism occurs by removing the load during torsional loading.

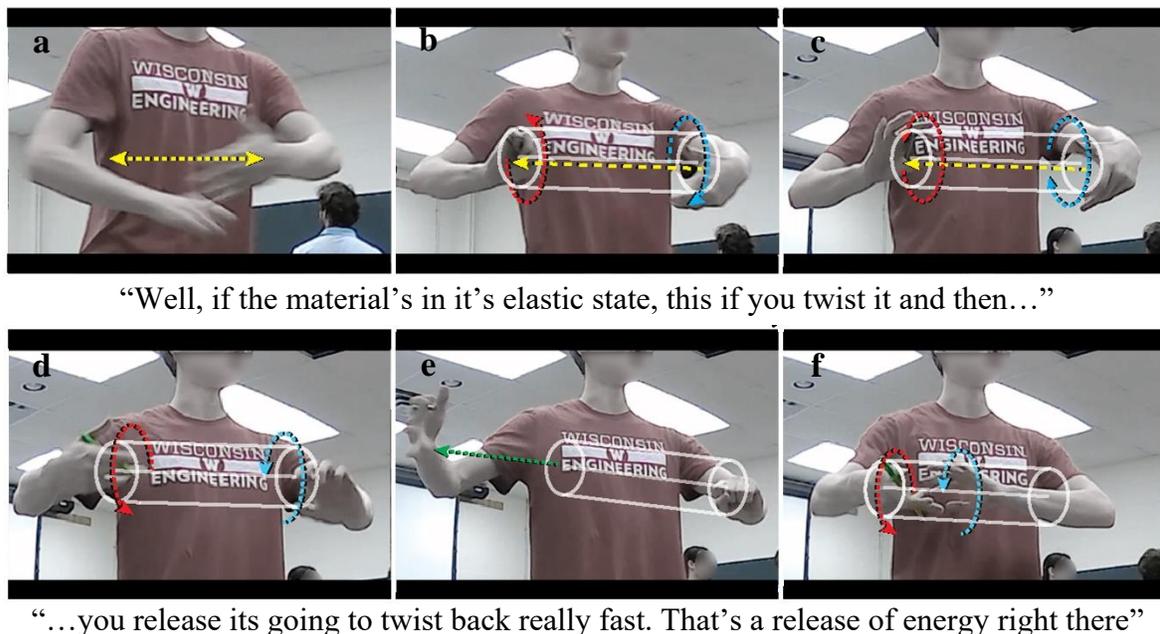


Figure 3: Depicts a student using a gesture sequence while describing energy addition and release during torsional loading. (a) the student spatially constructs a testing sample through a *geometrical shape* gesture. (b-d) the student uses the *rotational (loading)* gesture while describing deformation. (e) the student uses a *release* gesture to depict removal of the torsion load. (f) the student uses a *rotational (unloading)* gesture while describing how the material responds to the removal of the torsional load and energy release.

Figure 4 depicts a student making a concordant gesture while explaining where maximum shear stress occurred on a torsionally tested sample. The student's *geometrical shape* gesture (circular in shape) is concordant with their speech, "Maximum shear stress occurs on the outer radius [Geometrical Shape gesture]," correctly identifying that the largest torsional shear stress occurs farthest from the neutral axis at the outermost boundary layer or radius.



"Maximum shear stress occurs on the outer radius."

Figure 4: Depicts a student using a concordant *geometrical shape* gesture while describing the location of maximum shear stress.

In addition to the concordant gesture and co-speech matches, students also generated discordant gestures that did not match their speech. As a reminder, discordant gestures occur when verbal and nonverbal processes are operating differently; this can occur when: (1) verbal explanations correctly convey concepts, but gestures do not convey the same information, (2) when accurate gestures are misaligned with incorrect verbal explanations, or (3) when they are both correct or incorrect, but for different reasons.

Figure 5—on the next page—shows a student's verbal processes were correct, but one of their gestures expressed the concept incorrectly. In this example, the student is describing how a material responds to torsional loading, "I guess we can also add that like example of say twisting a rubber band." Here, the student used the metaphor of twisting a rubber band to describe how the material displaces and what occurs after the torsional load is released. Next, they follow up with a *rotational (loading)* gesture while depicting angular displacement, "You can see that [Rotational (Loading) gesture, figure 5.a-c] the... the material of a rubber band is [Rotational (Loading) gesture, figure 5.d-f] twisted up." Following this explanation, the student uses the *geometrical shape* gesture to simulate a sample for testing, "It's where [Geometrical Shape gesture, figure 5.g-i] you'd really see..." However, it is during this portion of their explanation that their speech and gesture becomes discordant. The student uses the rotational (loading) gesture to describe the material returning to previous, unloaded configuration, "...you can tell that it returned [Rotational (Loading) gesture, figure 5.j-l] to its original position." However, a continued load could permanently deform the material. In this case, the material would not return to its original configuration as described by the student.



Figure 5: Depicts a student’s gestural sequence while describing, through a rubber band metaphor, how a material responds during torsional loading. (a-c) the student uses a concordant *rotational (loading)* gesture to depict angular displacement. (d-f) the student using a concordant *rotational (loading)* gesture to depict the twisting portion observed during torsional loading. (g-i) the student uses a concordant *geometrical shape* gesture to spatially represent the material. (j-l) the student discordantly using a *rotational (loading)* gesture while describing the material returning to its original configuration.

Figure 6 depicts an instance where a student's gestures correctly conveyed that torsional loading causes a stress material response, but their verbal explanation was incorrect. The student says "...you can see how the material responds, depending on the material you can physically see... a... a... [(Rotational (Loading) gesture)] a... is that a transverse shear? Is that technically what it is?" However, in pure torsion, materials do not encounter transverse shearing but exhibit torsional shearing, making the student's verbal response conceptually inaccurate even though their gestures correctly simulated the behavior of the sample.



"...depending on the material you can physically see... a... a... a... is that a transverse shear?"

Figure 6: Depicts a student's discordant *rotational (loading)* gesture while describing shear.

Lastly, there were instances where student explanations—both verbal and gestural—were matched, but the student exhibited a conceptual misunderstanding of differences between ductile and brittle fracture mechanisms. A student discussing fracture mechanisms to their partner described a brittle fracture as “parallel” while generating a concordant parallel gesture, “...if it is a brittle material then it will fracture [*fracture, rotational (loading), and rotational (unloading) gesture*] and there, there'll be a very like parallel [*parallel gesture*] fracture point... whereas, like a more ductile material sort of bends [*fracture, rotational (loading), and rotational (unloading) gestures*] less.” The student's explanation that ductile materials bend less is incorrect since a sample under pure torsion does not bend, rather it rotates about a central axis. Their explanation of brittle and ductile fractures, “... as for a more ductile... or for a more brittle, sorry, material, it would be very flat [*parallel gesture*] fracture” incorrectly referred to the fracture mechanisms of brittle and ductile materials since brittle materials under torsion exhibit an approximate 45-degree fracture along a line of rotation in contrast to ductile materials that exhibit a relatively smooth, parallel-like fracture.

Discussion:

This exploratory investigation was motivated by the need to understand whether engineering students' developing knowledge of torsion is exhibited through gestures (RQ1) and the value that is provided by analyzing these gestures for describing their reasoning and learning (RQ2). Video analyses of students' speech and gestures showed that students indeed enact a multitude of gestures during their explanations with classmates (as shown in Table 1). In this section, we attempt to broaden the perspective that this early study has to offer to scientific accounts of learning and practical implications for improving education, while acknowledging the many limitations of this one study in the context of the broader aims of engineering education.

An initial concern is that engineering knowledge organized primarily around symbolic formalisms may marginalize some forms of students' emerging knowledge. Gesture as a mode

of explanation that conveys a person's knowledge state and meaning making is well documented [3, 13,14]. Gestures are frequently used in engineering learning settings by both students and their instructors to convey ideas about physical and mathematical phenomena [18]. One powerful way this is done is through simulated action [12]. We found that some of the gestures that were observed reveal ways that students engage their bodies as epistemic resources in order to simulate a material's physical response while undergoing torsional loading. Gestures reveal some of the embodied ways that students simulate the material response and support their reasoning [12] about torsional loading concepts.

For example, a material experiencing torsion rotates about its central axis. Students conveyed this concept through the rotational (loading) and rotational (unloading) gestures. Students frequently used the geometrical shape gesture to simulate the material's geometry and location of shear stresses (shape-rod, cross-sectional geometry-circle, and location of the maximum stress-circular). The magnitude (increase) gesture enacted the increase in magnitude of shear stress that develops from the central axis towards the outermost boundary layer of the torsional sample. Gesture was also used to make novel inferences. A student deduced from their gesture and speech that energy is related to work (i.e., the change of length due to an acting force) and the more the outer arc length of sample deformed, the more energy was stored in the outermost boundary layer which would experience the largest stress. These nonverbal forms of explanation complement a student's verbal reasoning during problem-solving activities. These findings add to a growing repository of findings that gestures can facilitate learning in STEM [15-17].

Gestures express spatial and causal reasoning through movement in ways that complement what is not easily expressed through speech. Discordant gestures add information about learners' cognition by incorporating gesture information is expected to provide a richer and more complete assessment of students' emerging knowledge that can be used to inform instruction.

During generation of discordant gestures, the learner is in a state of two minds, one described through verbal processing and the other depicted through gesture. Discordant gestures index a learner's transitional state of knowledge when a learner is at a point of learning new concepts or reconceptualizing preconceived ways of thinking [19]. One example in this data set showed how a student verbally expressed an elastic regime (a rubber band metaphor) to describe how a material responds under torsional loading. However, the students' contemporaneous gestures (in this case, *rotational (loading)*) revealed that they also depicted the material straining into the plastic regime. A second case illustrated how a student may have the correct idea expressed through gesture (torsional shear stress) while at the same time generating an incorrect verbal explanation (transverse shear stress, which occurs due to bending). Across a range of discordant cases (i.e., incorrect gestural depictions, verbal descriptions, or when they are both correct or incorrect, but for different reasons), there is evidence that students are simulating [12] the material responses and fracture mechanisms of a sample undergoing torsional loading.

In addition to insights about engineering students' reasoning processes, these findings offer implications for improving engineering education. One implication is the role of students' gestures in formative assessment practices. Formative assessments are intended to provide situated opportunities for identifying students' reasoning and for providing responsive instruction

to enhance one's understanding or provide corrective interventions to avoid or correct emerging misconceptions. In addition, tracking student gestures during laboratory work and explanations can also reveal when students are in transitional knowledge states and are highly receptive to new instruction. In particular, discordant gestures indicate a students' heightened readiness to learn [21] and can signal opportunities for instructors to enhance the learning experience.

Gestures also reveal the importance of eliciting and cultivating embodied learning experiences for students. Highly math-centric approaches to engineering education do little to engage the powerful and flexible resources for grounding and embodied simulation that are available to all learners.

Finally, this study has several limitations. The sample of size of this pilot study is small and the students were focused on a limited set of physical phenomena. This provided a proof-of-concept that is now informing the design of a larger study with students from multiple class sections to provide a far richer data set of the varieties of ways that students engage their bodies during intellectually demanding activities. We also relied on the social interactions among students and the interviewer to generate the data. Such exchanges are rare during educational assessment practices. While a limitation for its fit to traditional educational practices and lecture-based educational environments, social contexts of these kinds that elicit explanation and simulated action are commonplace in the workplace and suggest that engineering programs consider expanding the array of assessment designs in use to foster greater production. Assessments of this kind rely on data generation using recorded video from which engineering instructors can isolate instances of gesture use-concordant and discordant-when students reason about engineering concepts; completed post hoc once the teaching intervention has concluded. However, once common gestures are isolated, engineering instructors can leverage this gesture repository during instruction to ground student knowledge in bodily actions.

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