Connecting Mathematics, Spatial Ability and Spatial Anxiety

Kelsey E. Schenck, University of Wisconsin-Madison
Mitchell J. Nathan, University of Wisconsin-Madison

Abstract: This study provides evidence that spatial ability and spatial anxiety impact mathematics ability. These findings (N = 101) are consistent with theories of how spatial ability and anxiety and add to the understanding of spatial ability's effect on mathematics. Three key findings include: 1) spatial ability and math ability are highly correlated; 2) specific spatial sub-categories could be more critical for success on different types of math tasks, and 3) high spatial anxiety scores predict lower spatial and math ability scores. Taken together, these results reflect the complex nature of spatial and math ability. Moreover, this research provides the first step in identifying and understanding the deeper relationships between math ability, spatial ability, and spatial anxiety.

Keywords: Mathematics, Spatial Cognition, Spatial Anxiety

Objectives/Purpose

Traditional models of mathematical cognition (e.g., Newell & Simon, 1972) have seen substantial expansion beyond individually isolated rule-based processing of symbol systems. While contextual and social influences on mathematical thinking have greatly influenced contemporary theories (e.g., Cobb, 1994; Schliemann & Carraher, 2002), two other classes of processes that extend our theories of mathematical cognition – embodiment and affect – are still in their infancy. One facet of embodied cognition particularly relevant for math is spatial skills (Uttal & Cohen, 2012; Mix et al., 2016; Newcombe, 2013). An aspect of affect relevant to math is anxiety. To further understand the breadth and multifaceted nature of mathematical thinking in terms of expanded notions of cognition, we investigate the relationship of mathematical ability with spatial skills and spatial anxiety. Models of mathematical cognition that include affective and embodied processes contribute to basic theory of human cognition. Such models also inform the design of evidence-based educational practices, including the design of curriculum activities, and principles for instruction and assessment.

Theoretical Framework

Spatial Ability

Spatial ability refers to the ability to generate and manipulate spatial objects, images, relationships and transformations (Battista, 2007). These abilities include memorization and comparison of visual patterns, manipulating mental objects, and performing spatial imagery (Hegarty & Waller, 2005). Spatial imagery, as defined by Hegarty & Waller (2005), refers to "a representation of the spatial relationships between parts of an object, the location of an object in space or their movement" (pg. 144). Spatial ability has been linked to success in mathematics for students as young as three (Verdine et al., 2014), and studies investigating phenomena such as the SNARC effect have the links between spatial-numerical associations and math ability (Berch, Foley, Hill & Ryan, 1999; Toomarian, Meng, & Hubbard, 2019). As children develop, spatial ability is consistently important. Assessment of spatial skills among elementary-aged children strongly predict later mathematical capabilities (Casey et al., 2015; Laski et al., 2013).

There is little consensus in the literature about the exact combination of factors and sub-skills critical for spatial ability (Yilmaz, 2009), and naming conventions of the factors vary (McGee ,1979; Lohman, 1988; Hegarty & Waller, 2005). We adopted the three-factor framework proposed by Ramful and colleagues (2016): (1) *Mental rotation* describes how one imagines a 2D or 3D object would appear after it has been turned; (2) *spatial orientation* involves egocentric representations of objects and locations and includes the notion of perspective taking; and (3) *spatial visualization* describes mental transformations that do not require mental rotation, spatial orientation, or egocentric reference.

Anxiety in Mathematics and Spatial Reasoning

Though some scholars argue that low levels of general anxiety are essential for neurocognitive performance, high levels of anxiety reduce neurocognitive performance (Derakshan & Eysenck, 2010; MacLeod & Donnellan, 1993; Meyers, Grills, Zellinger, & Miller, 2013). Mathematics anxiety predicts low mathematics performance even when controlling for other anxiety factors such as test anxiety (Lukowski et al., 2016). Furthermore, math anxiety appears to be domain specific: When students simply anticipated doing math, those with high math anxiety exhibited greater activity in brain regions associated with threat detection, which was not present when they anticipated doing a reading activity (Lyons & Beilock, 2012).

Spatial anxiety as a psychological construct captures feelings of annoyance, confusion, and frustration when faced with spatial tasks. Spatial anxiety and spatial ability are negatively correlated (Malanchini et al., 2017). Furthermore, spatial anxiety, like spatial ability, appears to be composed of sub-components. One study of twins identified two components of spatial anxiety: navigation and rotation/visualization (Malanchini et al., 2017), or large versus small scale (Hegarty, Burte & Boone, 2018). Other studies have linked spatial anxiety to spatial orientation skills, specifically, a decrease in efficiency of orientation strategies and an increase of errors on navigation tasks (Lawton, 1994; Hund & Minarik, 2006). However, we know little about the relationship of spatial anxiety on math performance.

Research Questions

There is emerging evidence for connections between the broad constructs of spatial ability, spatial anxiety, and math performance, overall, but little is known about how spatial anxiety relates to math performance, or the associations for their various subcomponents. Thus, our three main research questions are: What is the relationship between spatial ability and overall performance on math tests? (RQ1); What are the relationships between spatial ability sub-categories and performance on math performance sub-categories? (RQ2); What are the relationships between spatial anxiety, spatial ability, and performance on math tests? (RQ3).

Methods and Data Sources

We recruited 153 participants (18 years of age and above) through Amazon's Mechanical Turk service. All participants completed spatial ability, spatial anxiety, and mathematics ability assessments, and a demographics survey. For spatial ability, participants completed the *Spatial Reasoning Instrument*, which breaks down into three sub-categories: mental rotation, spatial orientation, and spatial visualization (Figure 1; Ramful et al., 2016). The spatial anxiety measure was a combination of 8 questions from the *Spatial Anxiety Scale* (Lawton, 1994) and 5 questions from the *Child Spatial Anxiety Questionnaire* (Ramirez et al., 2012) that were updated to fit the population. The mathematics ability measure was composed of a subset of 16 questions from the 2012 PISA Mathematics Test, which consisted of 4 sub-categories of questions: quantity, uncertainty and data, space and shape, and change and relationships (Organization for Economic Co-operation and Development, Programme for International Student Assessment, 2014).

All measures were completed online. In a previous pilot study, participants took an average of 26 minutes to complete the tasks (Author, 2019). Based on this data, participants who completed the study in less than 25 minutes were excluded from the analyses (Figure 2). This restriction left a total of 101 participants. See table Tables 1 and 2 for descriptive statistics.

Results and Discussion

Reliability for the spatial ability, mathematics, and spatial anxiety measures exceeded the 0.7 threshold for the overall assessments and for each of the sub-scales, with the exception of the mathematics Quantity sub-scale, which had a Cronbach's $\alpha = 0.69$ (see Table 3 for further details).

For our analyses, we used Ordinary Least Squares (OLS) regression. In each model, we included participant sex and age as fixed effects, because these variables have been shown to be significant predictors of spatial ability, spatial anxiety, and mathematics ability (Lawton,1994; Maeda & Yoon, 2013, Voyer, Voyer & Bryden, 1995).

RQ1: Spatial and Math Ability

A multiple linear regression was calculated to predict overall math ability scores based on overall spatial ability scores while controlling for age and sex (RQ1). Spatial ability scores significantly predicted overall math ability scores (t (97) =10.07, $p < .001$). Each one-point increase in spatial ability scores increased math ability scores by 0.38 points. This result indicates that spatial ability may have an important relationship with math ability and gives further weight to previous findings that spatial ability may be an essential factor for success in STEM fields (Davis, 2015; Uttal & Cohen, 2012).

RQ2: Spatial and Math Ability Sub-categories

To investigate the relationships between spatial and math ability sub-categories, we calculated five separate multiple linear regression equations to predict math ability scores based on each of the spatial ability sub-categories (mental rotation, spatial orientation, and spatial visualization) controlling for age and sex (RQ2). The first model predicted overall math ability by the three spatial ability sub-categories. The subsequent models predicted the four sub-categories of mathematics ability (quantity, uncertainty and data, space and shape, and change and relationships) by the three spatial ability sub-categories. Overall, at least one spatial ability sub-category significantly predicted the dependent variable. No sex effects were found in any model.

Mental rotation was significantly predictive of scores for both uncertainty and data questions (UD; t (95) = 2.84, p = .005) and change and relationship questions (CR; t (95) = 2.50, p = .014), but not math ability overall. Spatial orientation was significantly predictive of scores for overall math ability (t (95) = 3.50, p = 0.001), quantity questions (Q; (t (95) = 3.94, p < .000), and uncertainty and data questions (UD; t (95) = 1.99, p = .049). Spatial visualization was significantly predictive of scores for both uncertainty and data questions (UD; t (95) = 2.18, p = .031) and space and shape questions (SS; t (95) = 3.10, p = .003).

RQ3: Spatial Ability, Math Ability, and Spatial Anxiety

The last set of analyses further examined the relationships between spatial ability, spatial anxiety, and math ability (RQ3). Results indicated that spatial anxiety significantly predicted spatial ability scores (t (97) = -8.18, p < .000). As expected, this relationship was negative, which is consistent with previous work on the effects of anxiety on performance (e.g., Malanchini et al., 2017).

A second model indicated that spatial anxiety also significantly predicted math ability scores (t (97) = -0.13, p < .000). This result was interesting since the results of the pilot study did not indicate this relationship, and it has not been identified in any previous literature to our knowledge. The relationship between spatial anxiety and math ability is negative, as expected with anxiety relationships. Spatial anxiety may affect students' spatial ability, which leads to lower math ability scores. Spatial anxiety scores may reflect math anxiety, which, though not measured in this study, was not significantly correlated in the pilot study (Author, 2019).

To determine the more specific relationship between spatial ability, spatial anxiety, and math ability, the third model in these analyses included spatial ability scores, spatial anxiety scores, and the interaction between these scores. Here, spatial anxiety lost its predictive power for math ability once spatial ability was added to the model (t (95) = -1.26, p = .211). Spatial ability remained highly predictive of math performance (t (95) = 2.51, p = .0134). Additionally, the interaction between spatial ability and anxiety was not significant (t (95) = 1.40, p = .165). This result could mean that spatial anxiety does not directly affect math ability but may indirectly affect it through other factors.

Educational and Scientific Importance

This study indicated three important results. First, spatial ability and math ability are highly related. Second, specific spatial sub-categories could be more critical for success on different types of math tasks. Third, high spatial anxiety scores predict lower spatial and math ability scores.

Taken together, these results reflect the complex nature of spatial and math ability. This research helps uncover the deeper relationships between math ability, spatial ability, and spatial anxiety. The strong correlation between spatial ability and math ability may reflect the highly spatial nature of mathematics.

Additionally, the results illuminate the relationships between the different sub-categories of spatial and math ability that previous research has not identified. Each of the three spatial ability sub-categories (mental rotation, spatial orientation, and spatial visualization) had unique relationships with the four sub-categories of math ability. Spatial visualization was predictive of success on items involving space and shape, suggesting people visualize different views of an object without the need to rotate the object or orient relationships between objects. These differing relationships are consistent with the idea that spatial ability is composed of a variety of factors (e.g., McGee, 1979). Subcomponents of spatial ability factors may be more critical to success on specific math ability tasks than overall spatial ability. Thus, it may be possible to design more specific spatial interventions to improve scores on particular math sub-categories.

Spatial anxiety was negatively associated with both spatial ability and math ability. Spatial anxiety may have a general effect such as reducing working memory capacity, as posited in Attentional Control Theory (Eysenck &

Derakshan, 2011), which would limit resources devoted to mathematical problem-solving. Spatial anxiety may also disrupt specific mathematical skills.

To address the limitations of this study, we suggest three additional studies to explore these findings. First, we advocate the addition of verbal and spatial working memory tests, and biometric measures of anxiety that go beyond self report. Second, we wish to extend the participants as a large number of participants were white and spoke English as a first language.

References

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester Jr. (Ed.), Second handbook of research on mathematics teaching and learning (pp. 843-908). Charlotte, NC: Information Age Publishing

Berch, D. B., Foley, E. J., Hill, R. J., & Ryan, P. M. (1999). Extracting parity and magnitude from Arabic numerals: Developmental changes in number processing and mental representation. Journal of Experimental Child Psychology, 74, 286–308.

Casey, B. M., Pezaris, E., Fineman, B., Pollock, A., Demers, L., & Dearing, E. (2015). A longitudinal analysis of early spatial skills compared to arithmetic and verbal skills as predictors of fifth-grade girls' math reasoning. *Learning and Individual Differences*.

Cobb, P. (1994). Where Is the Mind? Constructivist and Sociocultural Perspectives on Mathematical Development. *Educational Researcher*, *23*(7), 13–20.

Davis, B. (Ed.). (2015). Spatial reasoning in the early years: Principles, assertions, and speculations. New York, NY: Routledge.

Derakshan, N., & Eysenck, M. W. (2010). Introduction to the special issue: Emotional states, attention, and working memory. *Cognition and Emotion*, *24*(2), 189–199.

Eysenck, M. W., Derakshan, N., Santos, R., & Calvo, M. G. (2007). Anxiety and cognitive performance: Attentional control theory. *Emotion, 7*(2), 336-353.

Hegarty, M., Burte, H., & Boone, A. P. (2018). Individual differences in large-scale spatial abilities and strategies. In *Handbook of Behavioral and Cognitive Geography*. Edward Elgar Publishing.

Hegarty, M., & Waller, D. A. (2005). Individual Differences in Spatial Abilities. In P. Shah (Ed.) & A. Miyake, *The Cambridge Handbook of Visuospatial Thinking* (pp. 121-169). New York, NY, US: Cambridge University Press.

Hund, A. M. & Minarik, J. L. (2006). Getting From Here to There: Spatial Anxiety, Wayfinding Strategies, Direction Type, and Wayfinding Efficiency. *Spat. Cogn. Comput*. 6, 179–201.

Laski, E. V., Casey, B. M., Yu, Q., Dulaney, A., Heyman, M., & Dearing, E. (2013). Spatial skills as a predictor of first grade girls' use of higher level arithmetic strategies. *Learning and Individual Differences*, *23,* 123–130.

Lawton, C. A. (1994). Gender differences in way-finding strategies: Relationship to spatial ability and spatial anxiety. *Sex roles*, 30(11-12), 765-779.

Lohman, D. F. (1988). Spatial abilities as traits, processes, and knowledge. In R. J. Stenverg (Ed.). Advances in the psychology of human intelligence (pp. 181-248). Hillside, NJ: Erlbaum.

Lukowski, S. L., DiTrapani, J., Jeon, M., Wang, Z., Schenker, V. J., Doran, M. M., . . . Petrill, S. A. (2016). Multidimensionality in the measurement of math-specific anxiety and its relationship with mathematical performance. *Learning and Individual Differences*.

Lyons, I. M., Beilock, S. L. (2011). Mathematics anxiety: Separating the math from the anxiety. *Cerebral Cortex*, 22, 2102–2110.

MacLeod, C., & Donnellan, A. M. (1993). Individual differences in anxiety and the restriction of working memory capacity. *Personality and Individual Differences*, *15*(2), 163–173.

Maeda, Y., & Yoon, S. Y. (2013). A Meta-Analysis on Gender Differences in Mental Rotation Ability Measured by the Purdue Spatial Visualization Tests: Visualization of Rotations (PSVT:R). *Educational Psychology Review*, *25*(1), 69–94.

Malanchini, M., Rimfeld, K., Shakeshaft, N. G., Rodic, M., Schofield, K., Selzam, S., . . . Kovas, Y. (2017).The genetic and environmental aetiology of spatial, mathematics and general anxiety. *Scientific reports*, *7,* 42218.

McGee, M. G. (1979). Human spatial abilities: sources of sex differences. New York, NY: Preager.

Meyers, J. E., Grills, C. E., Zellinger, M. M., & Miller, R. M. (2013). Emotional distress affects attention and concentration: The difference between mountains and valleys. *Applied Neuropsychology: Adult*, *21*(1), 28–35.

Mix, K. S., Levine, S. C., Cheng, Y. L., Young, C., Hambrick, D. Z., Ping, R., & Konstantopoulos, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of Experimental Psychology: General*, *145*(9), 1206–1227.

Newcombe, N. S. (2013). Seeing Relationships: Using Spatial Thinking to Teach Science, Mathematics, and Social Stuides. *American Educator*, 26–40.

Newell, A., & Simon, H. A. (1972). Human problem solving. Oxford, England: Prentice-Hall.

Organization for Economic Co-operation and Development, Programme for International Student Assessment. (2014) PISA 2012 Released Mathematics Questions. Retrieved from Organization for Economic Co-operation and Development website: http://www.oecd.org/pisa/test/PISA%202012%20items%20for%20release_ENGLISH.pdf

Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2012). Spatial anxiety relates to spatial abilities as a function of working memory in children. *Quarterly Journal of Experimental Psychology*, *65*(3), 474–487.

Ramful, A., Lowrie, T., & Logan, T. (2016). Measurement of Spatial Ability: Construction and Validation of the Spatial Reasoning Instrument for Middle School Students. *Journal of Psychoeducational Assessment*, *35*(7).

Schliemann, A. D., & Carraher, D. W. (2002). The Evolution of Mathematical Reasoning: Everyday versus Idealized Understandings. *Developmental Review*, *22*(2), 242–266.

Toomarian, E.Y., Meng, R., & Hubbard, E.M. (2019). Individual Differences in Implicit and Explicit Spatial Processing of Fractions. *Frontiers in Psychology, 10(569).*

Uttal, D. H., & Cohen, C. A. (2012). Spatial Thinking and STEM Education. When, Why, and How? *Psychology of Learning and Motivation - Advances in Research and Theory*.

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., Newcombe, N. S., Filipowicz, A. T., & Chang, A. (2014). Deconstructing Building Blocks: Preschoolers' Spatial Assembly Performance Relates to Early Mathematics Skills. *Child Development*, *83*(3), 1062–1076.

Voyer, D., Voyer, S., & Bryden, M. P. (1995). Magnitude of sex differences in spatial abilities: A meta-analysis and consideration of critical variables. *Psychological Bulletin*, 117(2), 250.

Yilmaz, B. (2009). On the development and measurement of spatial ability. *International Electronic Journal of Elementary Education*, *1*(2), 1–14.

Tables and Figures

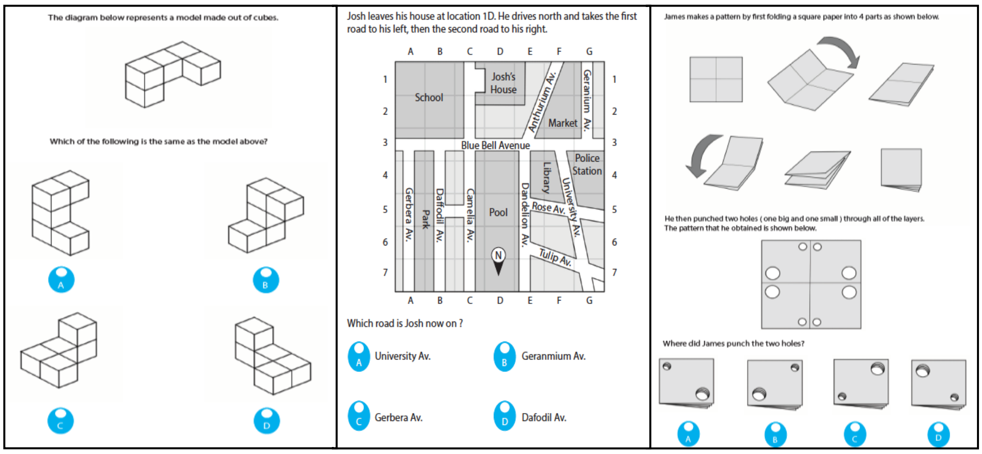

Figure 1. Mental Rotation (left), Spatial Orientation (center), and Spatial Visualization (right) questions taken from the *Spatial Reasoning Instrument* (Ramful, Lowrie & Logan, 2016)

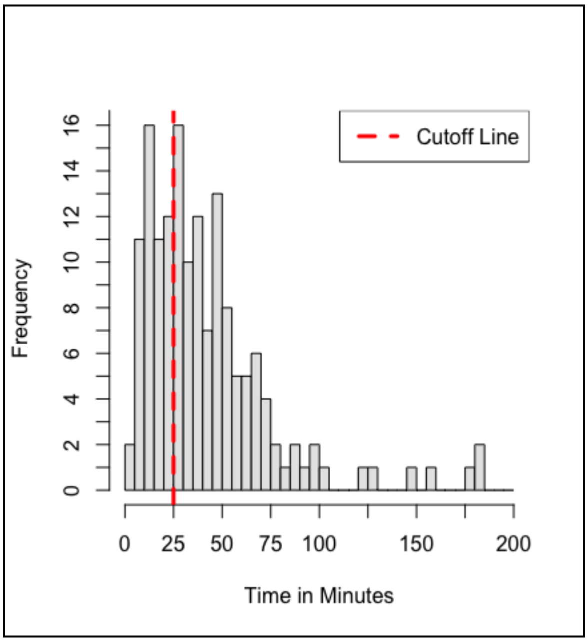

Figure 2. Participant Total Session Time

Table 1

Demographic Information (N = 101)

Variables	**Mean (SD)**	**N (%)**
Age in years	34.05 (10.97)	—
Sex, Female	—	62 (61%)
Native Language, English	—	86 (85%)
Ethnicity, White	—	63 (62%)

Table 2

Cognitive Statistics (N=101)

Variables	**Mean**	**SD**
Total SRI Score [30]	18.34	6.67
Mental Rotation (MR) Sub score [10]	5.89	2.73
Spatial Visualization (SV) Sub score [10]	4.58	2.60
Spatial Orientation (SO) Sub score [10]	7.87	2.22
Total PISA Score [16]	8.99	3.45
Quantity (Q) Sub score [4]	2.35	1.07
Uncertainty and Data (UD) Sub score [4]	2.31	1.14
Space and Shape (SS) Sub score [4]	2.37	1.05
Change and Relationships (CR) Sub score [4]	1.92	1.09
Spatial Anxiety Score [65]	23.44	12.29
CSAQ Sub Score [25]	15.21	5.33
SAS Sub Score [40]	8.23	7.71

Note. [*m*] reflects the maximum score.

Table 3

Reliability Checks for all Measures (N=101)

Variables	**Cronbach's** α
Total SRI Score	0.89
Mental Rotation (MR) Sub score	0.76
Spatial Visualization (SV) Sub score	0.72
Spatial Orientation (SO) Sub score	0.72
Total PISA Score	0.77
Quantity (Q) Sub score	0.69
Uncertainty and Data (UD) Sub score	0.71
Space and Shape (SS) Sub score	0.72
Change and Relationships (CR) Sub score	0.76
Spatial Anxiety Score	0.94
CSAQ Sub Score	0.86
SAS Sub Score	0.91

Note. All items appeared to be worthy of retention, resulting in a decrease in the alpha if deleted.