International Society of the Learning Sciences

Grounded and Embodied Proof Production: Are Gestures and Speech Enough to Produce Deductive Proof?

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Abstract: This study investigates the associations of spontaneous *dynamic gesture* and *transformational speech* with the production of *deductive proofs* in participants' reasoning about geometric conjectures (N=77). Although statistical analysis showed no significant association, the result suggests that purposefully including directed actions and pedagogical language in interventions could promote the production of deductive proofs.
Keywords: Geometry, Proof, Dynamic gesture, Transformational Speech

Nathan and Walkington (2017) developed the Grounded and Embodied Mathematical Cognition (GEMC) model (Figure 1) based on the Gesture as Simulated Action framework (Hostetter & Alibali, 2019) and the Action-Cognition Transduction hypothesis (Nathan, 2017). They showed how the activation of both the sensorimotor system and the language system is necessary, but perhaps not sufficient, for a learner to produce a mathematically *valid proof-with-insight*, a conceptual understanding of a conjecture that is correct, logical, and generalizable. The GEMC model posits that *dynamic gestures* and *transformational speech* – both the embodiments of simulated mental action – are reliable predictors of valid proofs-with-insight for mathematical conjectures (Nathan et al., 2020; Pier et al., 2019). *Dynamic gestures* (DG) depict transformational operations on mathematical objects (e.g., rotation or dilation; Figure 2). *Transformational speech* (TS) verbally articulates transformational operations (e.g., "You can increase or decrease the lengths of the sides and still have the same angles."). However, further investigation is needed on the extent of DG or TS's role in generating *deductive* mathematical proofs. Therefore, the research question for this study is: *When students produced valid proofs-with-insight, is there an association between their utilization of DG and TS and the production of deductive proofs?* A proof is *deductive* when it (a) is generalizable and holds for all cases under consideration, (b) utilizes clear logical inferences, and (c) exhibits goal-oriented thinking – where the prover progresses systematically with goals in mind and correctly anticipates the outcomes of the proposed operations (Harel, 2007). This study examined the role of DG and TS in the production of deductive proofs. We expect that the participants who utilized both DG and TS simultaneously while proving would produce more deductive proofs than those who did not.

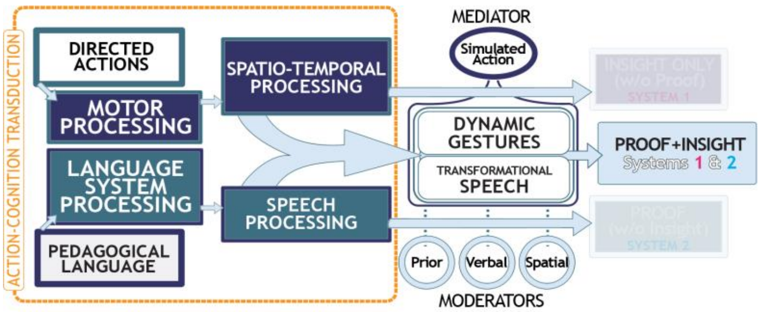

Figure 1. Grounded and embodied mathematical cognition (GEMC) model (Nathan and Walkington, 2017).

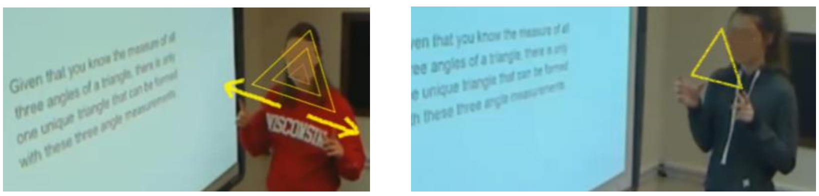

Figure 2. Dynamic gesture (left) and non-dynamic gesture (right).

This study is a post-hoc analysis of the data of 77 valid proofs-with-insight from college students who were recruited to prove four geometric conjectures. Participants were asked individually to prove the four conjectures projected on a screen. Figure 2 depicts students' gestures in response to one of the conjectures, 'Given that you know the measure of all three angles of a triangle, there is only one unique triangle that can be formed with these three angle measurements.' Their responses were video recorded and transcribed.

For this study, a *valid proof* was coded as a correct judgment on the given conjecture's truth value and *insight* as including key mathematical ideas of the conjecture. Then, each transcript of 77 valid proofs-with-insight was binarily coded (i.e., 0/1) in three categories: *DG, TS,* and *deductive*, then categorized into four groups by their utilization of DG and TS (Table 1). This random sampling of mutually independent observations satisfied the assumptions for a Fisher's exact test through which we tested whether there was an association between the utilization of DG and TS and the total number of deductive proofs produced.

Table 1: The contingency table of deductive proofs in the four groups by the DG/TS utilization

	DG & TS	**DG**	**TS**	**None**
Deductive	46	19	4	4
Non-deductive	2	1	0	1

Although the raw numbers seem favorable (Column 1, Table 1), statistical analysis showed no significant association between the DG/TS utilization and the production of deductive proofs ($p = .442$) and no significant difference in deductive proof production between the group who used both DG and TS and the groups who did not ($p = .629$). These null results contradict our hypothesis that DG and TS combined would induce more deductive proofs.

Nevertheless, we maintain that this anecdote is inconclusive to refute GEMC model for three reasons. First, multiple previous studies in the same line of research have shown significant associations of the concurrent occurrence of directed actions and transformational language with improved proving practices (e.g., Pier et al., 2019; Williams-Pierce et al., 2017). Second, the GEMC model posits DG and TS as necessary but not sufficient contributors. Third, the small sample size and low statistical power may have resulted in the high p-values. Consequently, we argue that our analysis suggests that spontaneous expression of transformation may be insufficient in generating deductive proof, a more rigorous proof form than valid proofs-with-insight. In turn, the result may hint at the need for interventions to include directed actions and transformational pedagogical language, as predicted in Nathan and Walkington (2017). Future research will investigate other possible contributors to deductive proof production.

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Acknowledgments

The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through Grant R305A160020 to University of Wisconsin-Madison. The opinions expressed are those of the authors and do not represent views of the IES or the U.S. Dept. of Ed.