Exploring Expanded Notions of Mathematical Reasoning: Spatial Systems, Anxiety, and Embodiment

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Abstract: This proposal seeks to investigate the relationship of mathematical ability with embodied systems: spatial skills and anxiety. Findings from preliminary studies are consistent with theories of how spatial ability and anxiety influence math performance. Proposed studies seek to expand on these findings using behavioral and neural evidence to explore these among members of a longitudinal cohort to identify embodied interventions that improve mathematical reasoning by increasing spatial engagement and reducing spatial anxiety.
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Goals and Background

While contextual and social influences on mathematical thinking have greatly influenced contemporary theories in mathematics education (Sfard, 2008), the role of embodied and affective processes are relatively underdeveloped. Spatial skills (Uttal & Cohen, 2012) and gesture (Alibali, 2005) are embodied cognitive processes particularly relevant for mathematics performance. Anxiety (e.g., Hund & Minarik, 2006) is an important affective process. There is also value in understanding how biological, cognitive and sociocultural processes combine to shape children's intellectual development and how such an understanding can inform the design of interventions for promoting math education. This study draws on the Grounded and Embodied Learning Framework (Nathan, in press), which describes the interconnected nature of learning across timescales, including processes that are biological (as measured by affective states and brain imaging), cognitive (spatial ability and number sense), knowledge-based (individual problem-solving), and sociocultural (gesture production, family demographics and household practices).

Prior studies link spatial anxiety to spatial orientation skills (Hund & Minarik, 2006). However, we know little about the relationship between sociocultural factors of one's development with spatial anxiety, gesture production, and math performance. To further understand the breadth and multifaceted nature of mathematical thinking in terms of expanded notions of cognition, we investigate the relationship of children's sociocultural and family demographics, mathematical ability with spatial skills and spatial anxiety. Viewing learning from this integrated, embodied developmental framework drives the following research questions: How are mathematical learning and thinking rooted in body-based processes and resources such as anxiety, spatial ability, and gesture production (RQ1), and how do they shape children's developing mathematical reasoning (RQ2)? How do embodied interventions influence mathematical thinking and learning (RQ3)? We use behavioral, neural and demographic data to explore these questions among members of a longitudinal cohort.

Methodology

Currently, we are collecting data from a longitudinal cohort of 2^{nd}-5^{th} graders (N=150) who are now in their fourth year (5^{th}-8^{th} graders). To address RQ1 and RQ2, participants in Study 1 complete extensive demographics surveys, mathematics, spatial ability, and spatial anxiety assessments, visual and verbal working memory tasks, and general intelligence tests. All assessments were refined in a pilot study to establish reliability and construct validity (N=101). Additionally, 10% of participants (n=15) will complete these assessments during fMRI sessions that will be used to explore functional changes in the brain that may link to spatial anxiety and spatial abilities. Spatial ability consists of: Mental rotation, spatial orientation (egocentric representations of objects and locations, including perspective-taking), and spatial visualization (nonrotational mental transformations, such as cuts and folds).

Correlational findings from Study 1 will inform the intervention design of Study 2. Participants (n = 80) will be selected based on demographics of those who demonstrate reliable relationships of anxiety to spatial and math performance, but who show low overall math performance. They will be prompted to produce gestures to investigate the hypothesis that gestures promote spatial reasoning (Alibali, 2005) while reducing anxiety (RQ3), as mediated and moderated by sociocultural factors, prior spatial processes and prior anxiety response levels.

Preliminary and expected findings

In a pilot study (N=101), we found that spatial abilities reliably predicted math ability but showed no sex differences (Schenck & Nathan, 2020). For example, children engage all three forms of spatial reasoning when reasoning about uncertainty and data, which may account for some of the challenges of learning to work with data. We also found evidence that spatial anxiety was reliably related to both math ability and spatial abilities.

In our proposed studies, we plan to replicate and expand the findings from our pilot study in a longitudinal, developmental framework. In Study 1, we expect to find neural evidence showing how different spatial skills and anxiety responses mediate and moderate mathematical reasoning, as moderated by demographic factors. Study 2 investigates whether prompted gesture production offers a novel avenue for improving mathematical reasoning by engaging spatial systems while not increasing anxiety. Furthermore, we will also investigate how neural correlates of anxiety responses may affect mathematical reasoning, perhaps by influencing the role of spatial systems shown statistically to moderate and mediate math performance.

Expected Contributions

The preliminary results reflect the complex nature of spatial and math ability. This research helps uncover the deeper relationships between math ability, spatial ability, and spatial anxiety across a range of ages and demographic groups. The strong correlation between spatial ability and math ability may reflect the highly spatial nature of mathematics.

Additionally, the results illuminate the relationships between the different sub-categories of spatial and math ability that previous research has not identified. Each of the three spatial ability sub-categories showed unique relationships with the four sub-categories of math ability. Spatial visualization was predictive of success on items involving space and shape, suggesting people visualize different views of an object without the need to rotate the object or orient relationships between objects. These differing relationships are consistent with the idea that spatial ability is composed of a variety of factors. Subcomponents of spatial ability factors may be more critical to success on specific math ability tasks than overall spatial ability. Thus, it may be possible to design more specific spatial ability interventions to improve scores on particular math sub-categories.

Spatial anxiety was previously found to be negatively associated with both spatial ability and math ability. Spatial anxiety may have a general effect, such as reducing working memory capacity, limiting resources devoted to mathematical problem-solving. Spatial anxiety may also disrupt specific mathematical skills. Thus, it may also be possible to design specific, age-appropriate spatial anxiety interventions to improve scores on mathematics assessments. Overall, these studies will add to the limited spatial anxiety literature and extend our knowledge about how affective and embodied processes impacts cognitive performance across a broad demographic range.

In our proposed studies, we explore associations between embodiment and mathematical reasoning via gesture production and spatial anxiety. Studies have shown that gestures and directed action predict learning and performance in mathematics (Cook, Mitchell, & Goldin-Meadow, 2008), as explained through the action-cognition transduction theory (Nathan, in press). This research is intended to provide an integrated behavioral and neuroscientific basis for understanding the development of mathematical reasoning and learning in terms of body-based systems that support spatiality, anxiety, and gesture production. It may also point toward the design of effective interventions based on embodiment for mitigating the effects of spatial anxiety on cognitive performance and academic growth.

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