EMBODIMENT AS A ROSETTA STONE: COLLECTIVE CONJECTURING IN A MULTILINGUAL CLASSROOM USING A MOTION CAPTURE GEOMETRY GAME

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The Hidden Village (THV) is a motion-capture video game for investigating how physical movements foster mathematical thinking and proof practices based on principles of embodied cognition. Analysis of the interactions of students in an all-Limited English Proficiency Title 1 high school geometry classroom revealed ways simulated enactment and collaborative gestural co-construction of mathematical ideas can bridge language barriers. These informed a redesign of THV to support both individual and collaborative play, as well as a collection of authoring tools for players to create their own content and upload it to an online database shared by users worldwide. Players, teachers and learners can implement custom directed movements that could foster deeper mathematical understanding and engagement for them and their peers.

Keywords: Technology; Reasoning & Proof; Geometry and Geometrical and Spatial Thinking

New technological interventions for learning mathematics are leveraging the embodied affordances of motion-capture technology to teach proportional reasoning (Abrahamson, 2015), algebraic reasoning (Ottmar et al., 2012), numerical training (Fischer et al., 2015), and geometric angles (Smith et al., 2014). As a design experiment (Brown, 1992), the development of *The Hidden Village* (THV) has been an iterative process of refining and extending its instructional application. THV is designed to help researchers better understand the grounded and embodied nature of geometric proof production by investigating how directed body actions, in combination with verbal prompts (i.e., pedagogical language), help students conceptualize the underlying mathematical ideas for geometric proof practices. Conducted in authentic classrooms, this type of *in situ* research exposes technologies like THV to dynamic environments in which "surprising occurrence[s]" emerge from students' collaboration and co-constructions to become sources for "ontological innovations" in the design process (diSessa & Cobb, 2004, p. 86). We present instances from gameplay of THV v.5, in which students renegotiated how the game was played, using a range of collaborative gestures and discussions to communicate shared understandings and present the newest revision, THV v.6.

Theoretical Background

Mathematicians use particular practices in their formulation of valid proofs. Research suggests that proof "is a richly embodied practice that involves inscribing and manipulating notations, interacting with those notations through speech and gesture, and using the body to enact the meanings of mathematical ideas" (Marghetis, Edwards, & Núñez, 2014, p. 243). Learners may derive new ideas and insights relevant to understanding and solving tasks based on their engagement in physical motions. Prior research shows that students' dynamic gestures reliably predict mathematical intuitions and proof validity (Nathan et al., 2018), even when controlling for spatial ability, gender, expertise, prior geometry knowledge, and speech content. Playing THV has been shown to help foster the production of beneficial gestures that promote higher proof performance (Authors, date). As study of collaborative proof has emerged, a growing body of evidence is demonstrating that collaborative gestures, as social extensions of cognition, are relevant to learner-learner interactions in the processes of mathematical sensemaking (Walkington et al., 2019). Collaborative gestures are physically and gesturally taken up by multiple learners. These co-constructive activities often extend cognition by echoing ideas, mirroring each other's reasoning, alternating in co-constructions and jointly operating in the same problem spaces (Walkington et al., 2019; *see* Table 1).

Table 1: Categories of Collaborative Gestures (adapted from Walkington et al., 2019).

Echo	Simple Echo (SE)	One learner makes a gesture and then a second learner makes the same (or a very similar) gesture afterwards.
	Echo & Build (E&B)	The second learner must change or add to the echoed gesture in some way.
Mirror	Simple Mirror (SM)	One learner makes a gesture, and then a second learner makes the same (or a very similar gesture) nearly at the same time.
	Anticipation (A)	One learner is gesturing, and then a second person anticipates a gesture they are about to do (correctly or incorrectly).
Alternate	Alternate & Build (A&B)	One learner gestures their understanding, and then another learner follows up, building upon or extending their reasoning.
	Alternate & Redirect (A&R)	One learner gestures their understanding, and then another learner follows up with a different gesture and reasoning.
Joint	(J)	Multiple learners manipulate mathematical object(s).

Such multimodal discursive practices in communicating mathematics (e.g., Edwards, 2009; Hall, Ma, & Nemirovsky, 2015; Radford, Edwards, & Arzarello, 2009; Roth, 1994, 2001) often externalize representations of students' minds and help establish and maintain *intersubjectivity* in a shared problem space (Nathan & Alibali, 2007). The design of THV draws from the theory of *Gesture as Simulated Action* (GSA; Hostetter & Alibali, 2019), the theory that people gesture because they activate perceptual-motor processes in the brain when they think about--and therefore simulate--the spatial or motoric properties of an idea while speaking and thinking. In this way, gestures can reveal the spatial and motor correlates of abstract and generalizable mathematical thinking. THV also draws on Nathan's (2014) model of *action-cognition transduction*, in which learners' movements serve as inputs that can drive the cognitive system into related cognitive states much like the cognitive system can, reciprocally, direct the motor system to make specific movements in response to one's thoughts and goals. It is this bi-directional process, in which cognitive states give rise to physical actions and vice-versa that THV is designed to demonstrate. As an embodied intervention, THV elicits movements (i.e., directed actions) from its players with the intent of influencing their cognitive processes in ways that fostering mathematical insights in support of the proof process. The action-based intervention from game play not only helps elucidate mathematical ideas for learners (Nathan, 2014; Walkington & Nathan, 2017) by engaging mathematically relevant simulations, but it also offers novel, embodied design opportunities for education researchers and practitioners.

In cases of collaborative gesture, *intersubjectivity* is a key determinant in the amount of collaboration and co-construction between actors. Figure 1 presents a 3D model of collaborative gesture ecology. As individuals collect into groups, intersubjectivity increases as group members echo, mirror, alternate and jointly gesture. Collaborative game play of THV can facilitate transduction to help students communicate their ideas about mathematics.

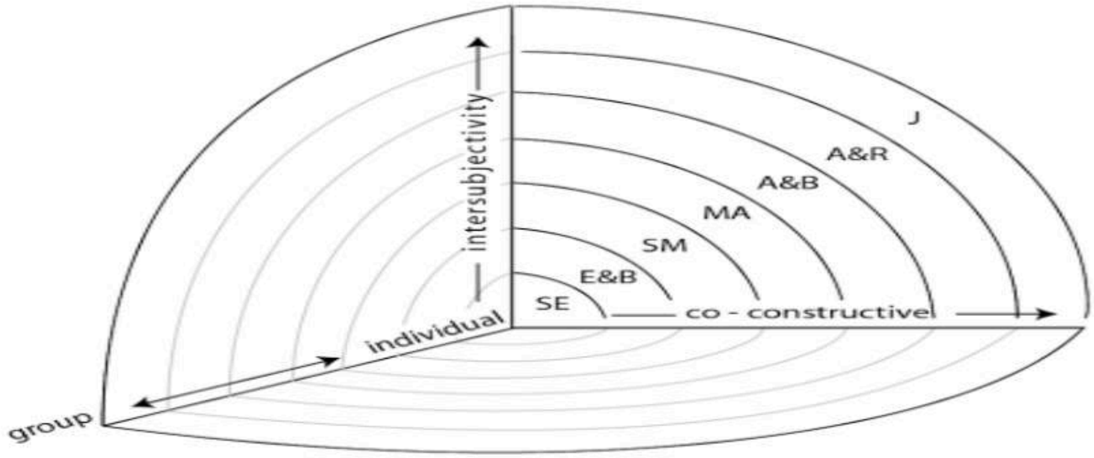

Figure 1: A 3D model of collaborative gesture ecology.

The Hidden Village

THV is a 3D motion-capture video game that delivers interactive math geometry curriculum in which each player mimics movements of in-game characters and then reads a geometry conjecture to determine if it always true or false. Each level of the game is comprised of 6 parts: Players meet members of the hidden village (Panel A), who implore players to perform movements (i.e., mathematically relevant directed actions detected by the MS Kinect; B). Next, players are given a math conjecture (C) and asked to indicate if the conjecture is true/false and why (D), followed by a multiple choice (E) and rewards and game achievement (F; *see* Figure 2).

Figure 2: The Hidden Village Game Play.

For example, in the triangle inequality conjecture (Figure 2, Panel C), a player will have performed the three movements (B) where they experience the arms extended laterally, then at an angle to the midline and then straight in front of the body. This series of movements is repeated 3 times. Next, they read the conjecture (C) and are prompted for an explanation (D). It is here that we video record their spontaneous representational and dynamic co-speech gestures that are hypothesized to contribute to their intuitions, insights and proof production. Then, players choose from among 4 multiple choices (E) before being rewarded by exposing a new portion of the Hidden Village map, a symbol, and energy strings to help power the ship (F).

Methods

Participants and Procedure

Over two days, we observed eight students in an all-LEP (Limited English Proficiency) Title 1 high school geometry classroom as they played THV v.5. Students' languages and ethnicities included Spanish from Central and South America, Arabic and French from North Africa, Hmong from Southeast Asia, and Chinese. Players were grouped as yoked pairs, alternating playing THV and observing game play of their partner.

Coding. Gameplay was audio and video recorded. The video clips were coded for whether participants made individual and collaborative gestures while validating the conjectures. Collaborative gestures were then coded by types identified by prior research (Walkington et al., 2019): echo, mirror, alternate, and joint (*see* Table 1).

Cases of Collaborative Gesture

Students, in light of the variability in English proficiency, dealt with the delivery of the game narrative, instructions, and mathematics in many ways, some unanticipated. In particular, many of the students used their bodies, objects in the room, and the bodies of other students to reason mathematically and formulate their justifications and proofs. To address comprehension and language production challenges, some students turned the dyadic game play into a collective activity where students contributed to each other's successful game play. They translated narratives and conjectures for each other and used directed actions to clarify and ground the math. Gestures traveled from the game to students and then crossed into different student groups as the movements simulated their mathematical ideas.

In this first transcript (*see* Figure 3), the students address the geometry conjecture: *The sum of the lengths of two sides of a triangle is always greater than the length of the third side*. In Figure 3 (below), students (S) engage in a discussion that leads to a series of collaborative gestures (panels A – F). Student 1 (S1, standing), turns to his partner, S2, to discuss the validity of the statement. S1 (*right*) listens to S2 (*left)* and *mirrors* (C) his gesture and then *builds* on the idea (D) in which S1 *anticipates* S2's gesture (E) before finishing *building* his explanation (F).

Figure 3: Embodied Collaboration – Mirror, Alternate and Build Gesturing.

S2: What are you doing? [Giggling]
S2: The sum of the length of two sides of a triangle is always greater than the lengths of the third side.
S1: The sum of the lengths of two sides of a triangle is greater than the length of the third side... No?
S2: Yes, because like the hypotenuse is the like, the greater one.

S1: Oh, yea, yea, yea.
S2: But if they add the other, the other side, then...
S1: The sum of the opposite angles...yea, so it is true.

In this next transcript (*see* Figure 4), S1 (*left*) and S2 consider the false conjecture: *If you double the length and width of any rectangle, then you exactly double the area*. S2 (a student from West Africa) explains the "doubling" of the sides to S1 (a student from Central America). S2's gestures extend a side (panel A), and S1 *echoes* the gesture (B). Then, S1 *builds* (C) upon the imaginary rectangle and that gesture is *echoed* by S2 (D), which is then *mirrored* by S1 (E), at which point they *alternate, build and redirect* gestures (F) as S1 gains insights about the relevant geometric relations.

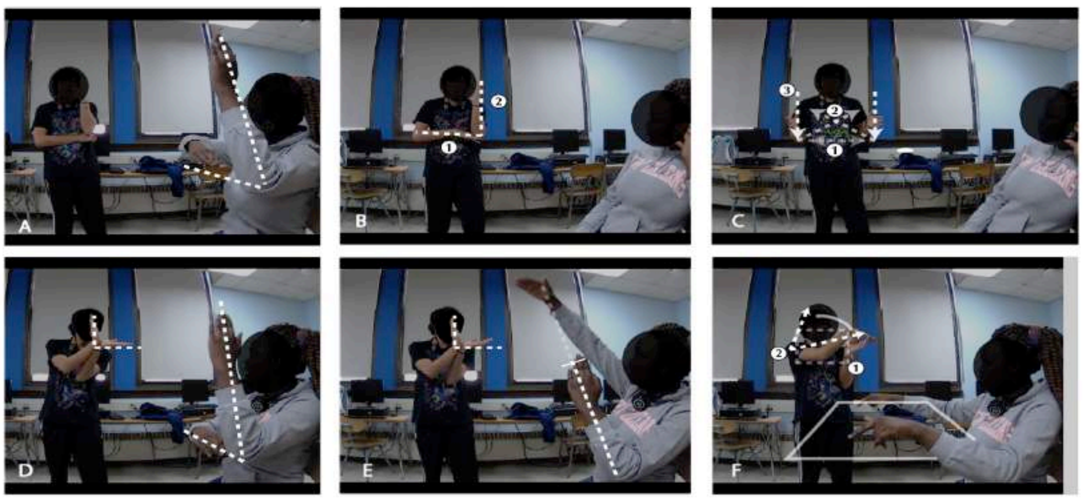

Figure 4: Embodied Collaboration – Echo, Mirror, Build, Alternate & Redirect Gesturing.

S1: So, the width of the rectangle... the area is...
S2: They, they multiplied it by 2 like if... if the length...
S1: So does it, does it have like, like this part?
S2: Let me show you the length – If the length was three
S1: Yeah
S2: They add another 3 and that becomes 6... If this one was 2, they add another 2 and that becomes 4.
S1: Oh...
S2: And they asking if the area, the area is like, like if that was a rectangle, the area is here, what is inside the rectangle?
S1: The angle?
S2: They asking if that would double, like if you...
S1: So... [Giggling]

In the final series (*see* Figure 5), S2 (*right*) asks for help translating words into Spanish and beckons S3 (*left*) over to interpret S1 (*center,* panel A). S3 translates S2's words in Spanish while *echoing* and *mirroring* the arm shape used by S3 (B & C) and replies "*Todo todo todo*" to S2 while adding several small circular motions between them (D). At this point, S3 *mirrors* S1's gestures (E) and then they gesture *jointly* (E) as they use a mutually shared discourse space. Finally, their discussion culminates in their convergence over a nearby tabletop to offload their ideas onto a workspace (F).

Figure 5: Embodied Communication – Echo, Mirror & Joint Gesturing.

S2: [Calls out to other Spanish speaking student for assistance]
S2: [to S3], can you explain to [S1] in Spanish what I mean? That, they double?
S3: Yea… What did it say?
S2: Like, here, this question, they double the length, and they double the, I don't know how to read in English...
S3: Oh.
S2: We talked about it... the width... [S1 and S2 engage in discussion in Spanish (Panels B – F)]

A Game for Collaborative Embodied Co-Creation

In effect, the students' collaborative co-constructive discussions while playing the original 8 conjectures in THV highlighted how varying levels of intersubjectivity combine to surpass language barriers and clarify concepts. Observations of students playing THV proved valuable for informing a redesign of THV. We learned that students may come up with novel movements that will help them understand the mathematical relations more clearly. Based on our observations, in the latest build of THV, students and teachers can now co-create the game characters' movements as ways to foster their own mathematical sensemaking and support the geometric reasoning of their classmates.

The conjecture editor allows players to co-create new levels of the game, including adding new conjectures and multiple-choice responses (*see* Figure 6). This content is stored and accessed via an external online database that allows users to share and play one another's content to support further collaborative, creative play, and learning. Additionally, players can collaboratively co-create new directed actions for a new conjecture using the Pose Editor (Figure 7), which is how THV "learns" to recognize new, user-generated poses. The process of creating and assigning movements is expected to deepen students' understanding of the embodied basis of geometric relations. When forming new poses (see Figure 7), each directed movement is comprised of 2-3 poses (starting, intermediate, and the target) that players design by posing the figure using pivots in the elbows and wrists. Once an individual pose is complete, players confirm or reset the figure. Once players have created all the poses (1), they can preview the sequence of directed action movements (2) in the form of a short GIF-like movie. Then can then set the matching tolerances (3) (i.e., % of allowable error for motion capture detection).

Figure 6: THV v.6 Conjecture Editor. Allows students to author their own content, including designing their own directed actions. Creating a New Conjecture (1) opens the Edit Conjecture (2) portal, where players name their team, name their conjecture, enter keywords, create a PIN, design a new icon, publish the conjecture to the online database repository, create their own directed actions, write the conjecture and devise 4 multiple choices for players to choose from and indicate which is correct.

A study that is currently underway in a set of ethnically and linguistically diverse, mixed ability high school geometry classes will assess how authoring conjectures and poses (i.e., directed actions) contributes to student learning. We hypothesize students making new content for the game will think, act, and talk through the ways that directed actions can foster mathematical insights and proof performance.

Figure 7: THV v.6 Pose Editor. The figure's limbs are posable via mouse movement. Figure can be rotated using a right-click drag or can be reset back to its origin (4). Being able to rotate the figure proved a critical for properly designing 3D directed actions.

Once students or teachers have created a collection of conjectures, they can cluster conjectures together in a game module (currently up to 8 maximum) for others to play (*see* Figure 8). Users can download individual conjectures or entire modules for their own use. As a tool for teaching, learning, and research, the conjectures within a module can be mixed and matched from any of the conjectures in the searchable database. Moreover, the module editor also allows the creator to customize a set of features for the module (e.g., offer hints, change the number of repetitions of the directed movements, etc.). This flexibility allows teachers to contour the game experience for their students. It also makes THV a flexible research tool. By making each feature of the game an option, investigators can create parallel versions of THV that only vary by a single variable, which is ideal for randomized controlled trials.

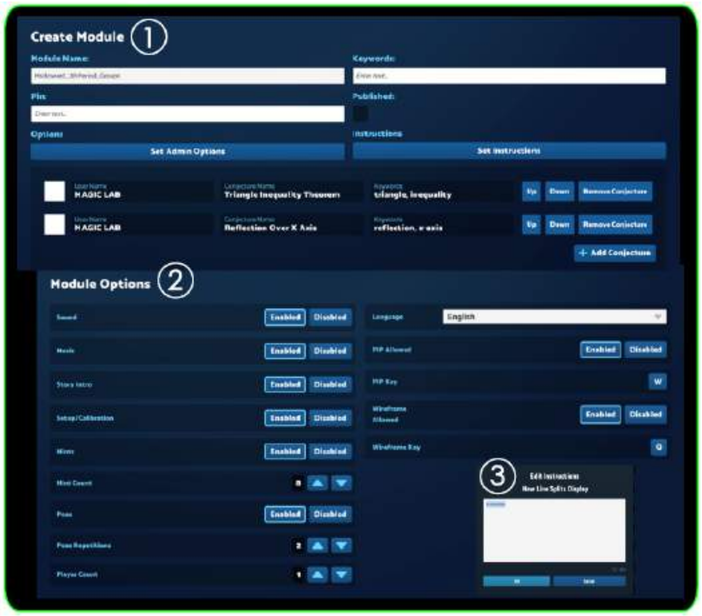

Figure 8: THV v.6 Module Editor. (1) Create a module, name the module, set the pin, customize the module (2) including: turning sounds, music, story, calibrations, hints, poses, number of players, language preferences, and scaffolding features. Players can add keywords (for easier search), publish to the database, and customize player instructions.

Discussion

Embodied principles of learning environment design offer some new opportunities for advancing students' mathematical reasoning. Embodied forms of reasoning offer a kind of Rosetta Stone that can bridge language barriers while supporting deep insights about the generalizable properties of space and shape. The co-construction of directed movements also allow students and teachers to institute more intuitive ways of embodying these mathematical ideas. The emergence of students' collaborative co-constructions inspired insightful ontological innovations in the redesign of THV. Our most proximate future goal is to elucidate how movements can embody spatial relations and transformations in geometry while empowering students to comfortably create their own content and instill confidence for deep, creative mathematics discourse. While THV currently focuses on high school geometry, the platform is applicable to other areas of mathematics that engage body-based forms of reasoning via simulation and metaphor (Lakoff & Núñez, 2000) as a path toward meaningful mathematical reasoning.

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