

What Makes Math Hard?

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and

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WCER

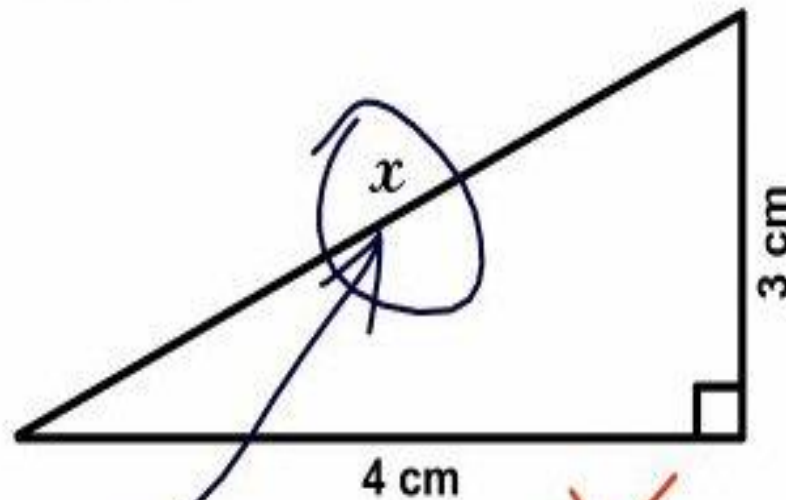


THE UNIVERSITY
of
WISCONSIN
MADISON

What Children Know and How They Know it



3. Find x .



Here it is

X

O

Mathematical Intuition



Lockhart's Lament

“Mathematics is not about erecting barriers between ourselves and our intuition, and making simple things complicated.

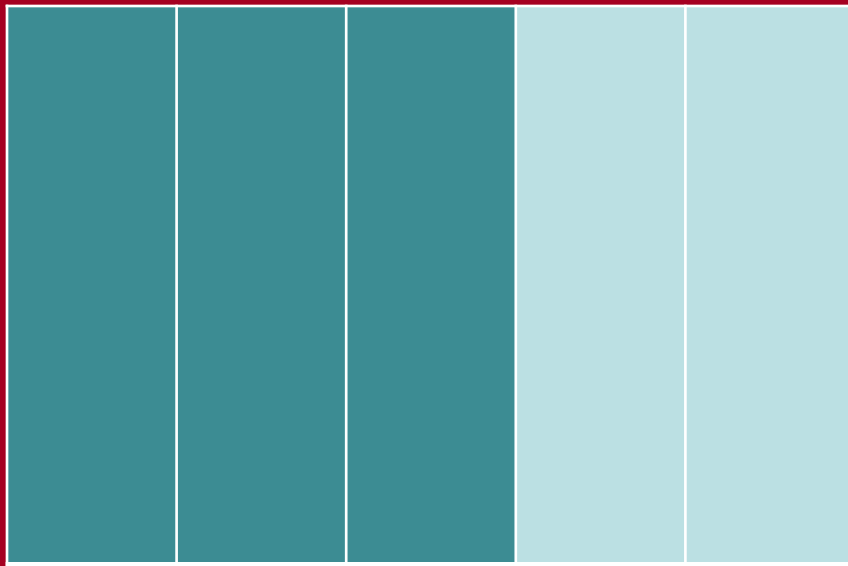
Mathematics is about removing obstacles to our intuition, and keeping simple things simple.”





$$\frac{5}{4} \div \frac{3}{4}$$

Visualize vs. Compute

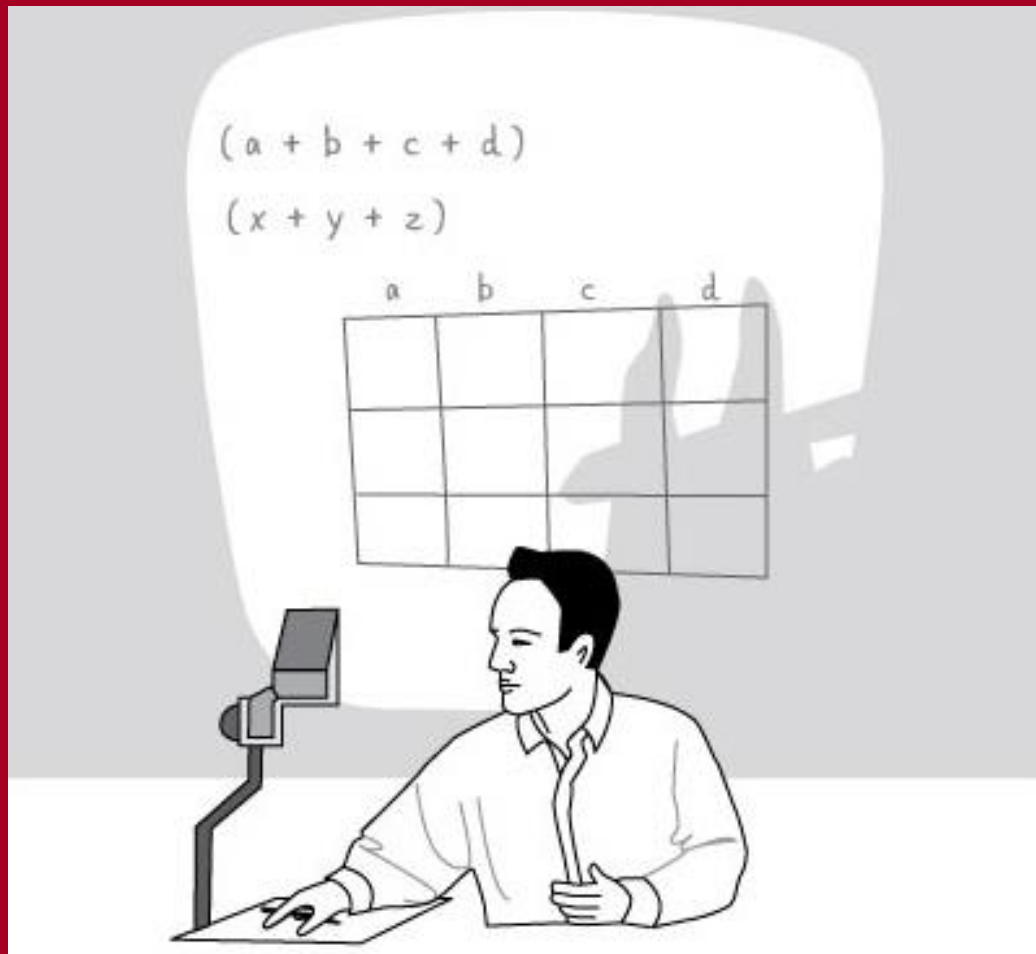


- A) $\frac{3}{5}$ of something
- B) $\frac{5}{3}$ of something
- C) $\frac{5}{3}$ of $\frac{3}{5}$
- D) $\frac{2}{3}$ of $\frac{3}{5}$
- E) $\frac{5}{4} \div \frac{3}{4}$



$$(a + b + c + d) \cdot (x + y + z)$$

Spatialize Algorithms





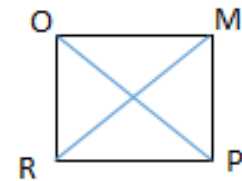
Geometry: Transform vs. Logic



The area of a parallelogram is the same as the area of a rectangle with the same length and height.

Given: ROMP is a square

Prove: $\overline{RM} \cong \overline{PO}$



Statement	Reasons
1. ROMP is a square	1. Given
2. $\overline{RO} \cong \overline{MP}$	2. Definition of square
3. $\overline{RP} \cong \overline{MO}$	3. Definition of square
4. $\angle R \cong \angle M$	4. Definition of parallelogram
5. $\triangle ORP \cong \triangle PMO$	5. SAS Congruence
6. $\overline{RM} \cong \overline{PO}$	6. CPCTC

PROOF 1

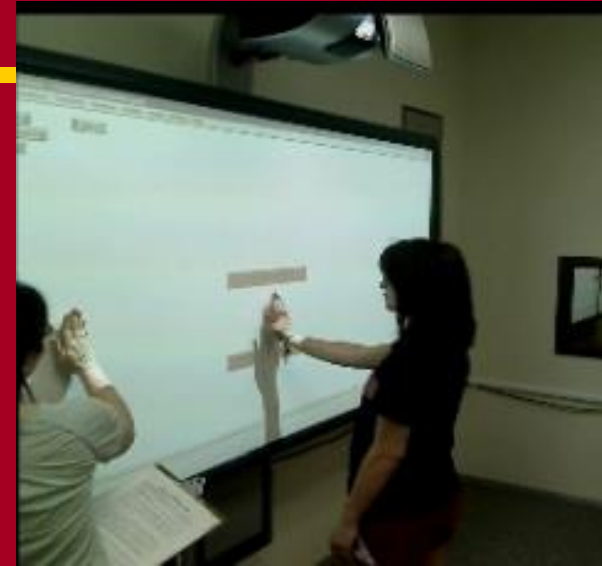
Geometry: Transform vs. Logic



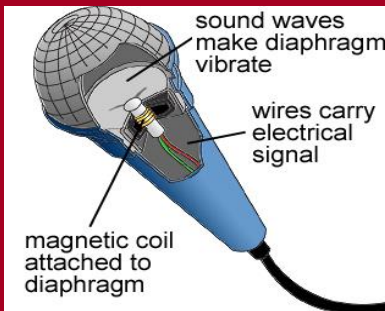
The area of a parallelogram is the same as the area of a rectangle with the same length and height.



Action Shapes Cognition

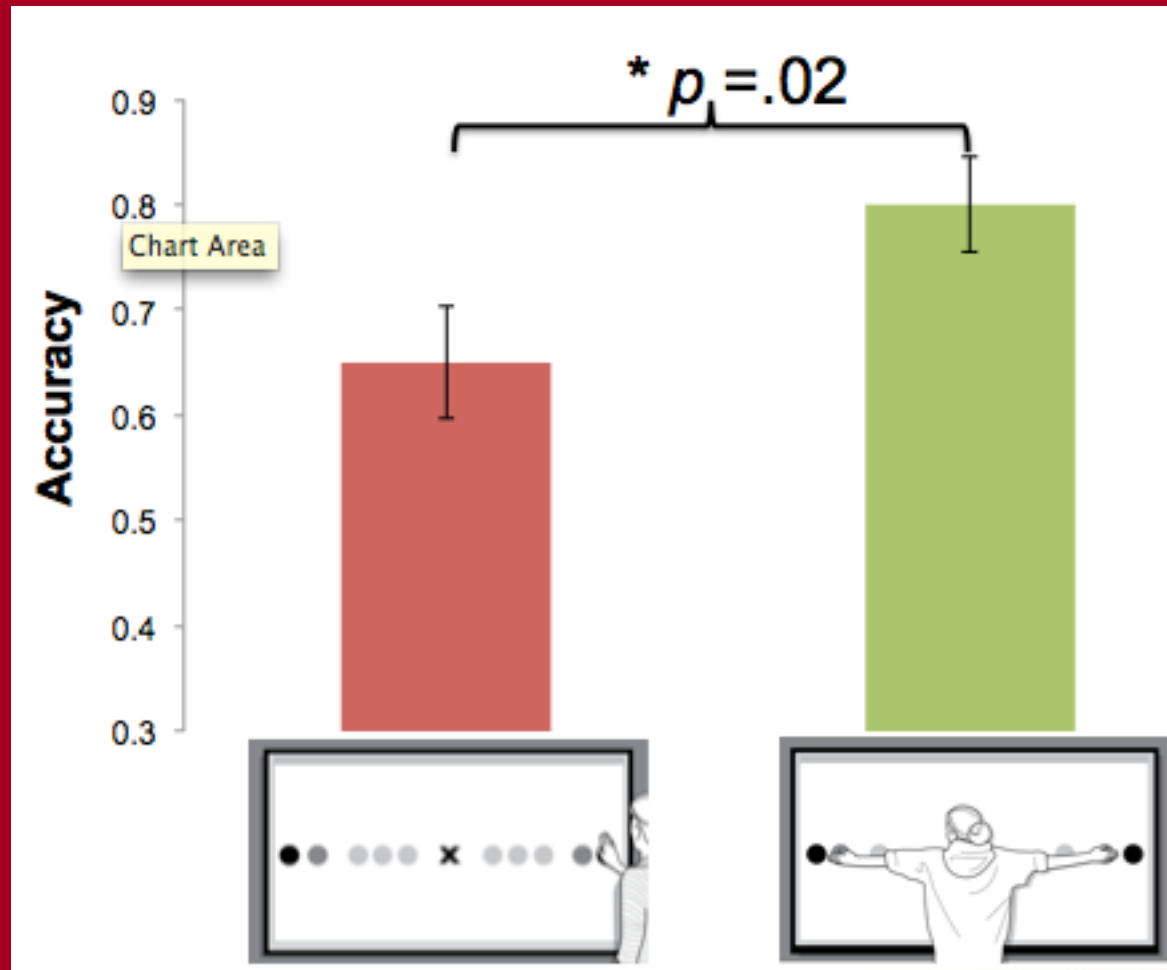


Transduction



Prior Work

Insight / Ascertaining

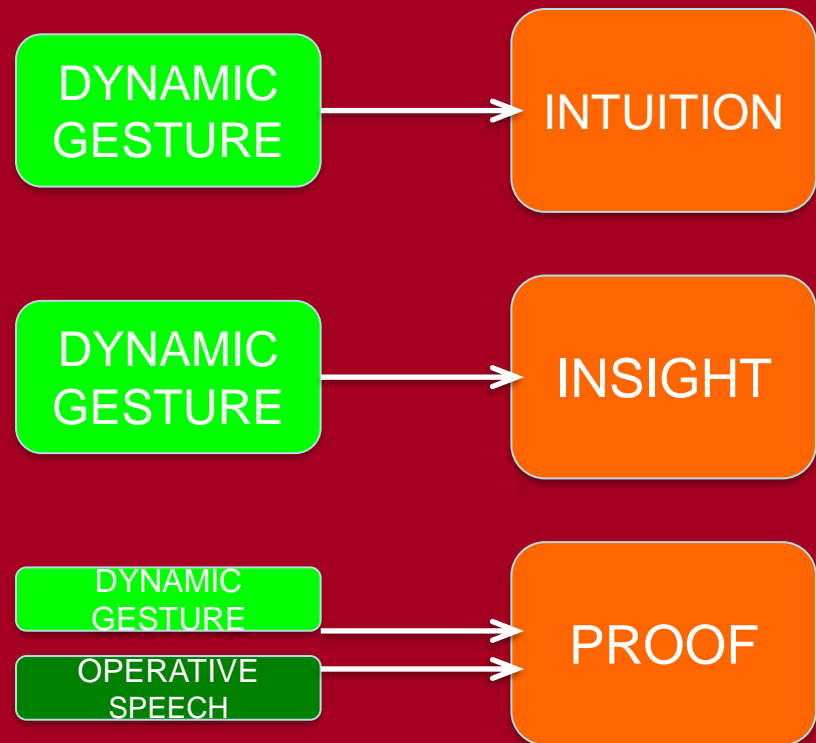


Nathan, Walkington, Boncodd, Pier, Williams & Alibali (2016)



Hypothesized Models

- Logistic Regression Models
- Random Effects: ID and Conjecture
- Fixed Effects:
 - Expertise (recruited)
 - Spatial-temporal reasoning
 - Verbal fluency
 - Ethnicity
 - Gender, Age, ELL status
 - Geometry knowledge

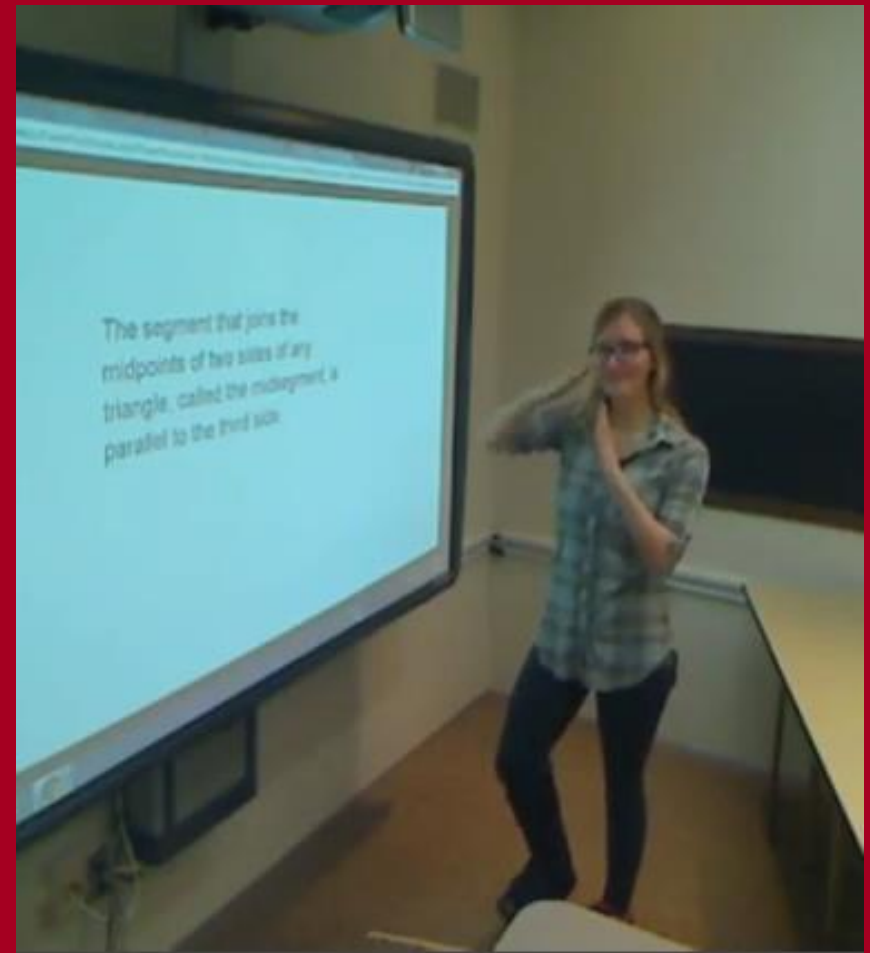




Mediating Variables

- **Dynamic gesture:**
Motion-based transformation of a mathematical object through multiple states

The segment that joins the midpoints of two sides of any triangle is parallel to the third side





Videos

Switch response after dynamic gestures

- [P002 Midsegment](#)

Comprehension error

- [P011 Conj1 Midsegment](#)

Borderline Compreh

- [P111 Conj6 Base Angles](#)

Dynamic gesture (expert)

- [P108 Circumscribe](#)

“False”

- [P120 Circumscribe](#)

Dynamic gesture “False”

- [P138 Midsegment](#)



Proof Model: w/o Gesture

Student ID
Conjecture

Random fx

Fixed fx

PROOF

Expertise: $d = 1.25$

Spatial: $d = 1.94$

Gender

Ethnicity

ELL status

Verbal fluency

Geometry knowledge



Proof Model: w/ Gesture



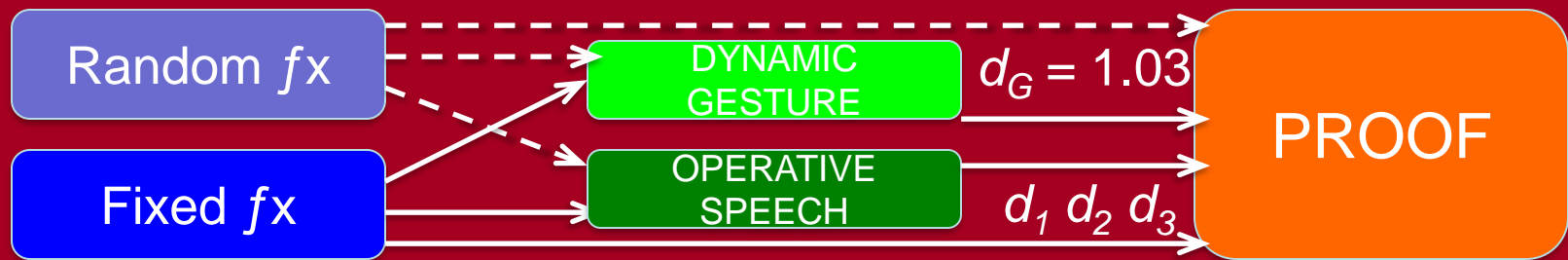
Expertise: $d_G = 1.26$

Spatial: ~~$d = 1.94$~~



Proof Model: w/ Gesture & Speech

Coh-Matrix 3.0



Expertise: $d_{G+OS} = 1.17$

Male: $d_{G+OS} = 0.60$

$d_1 = 1.50$ 1. *Intentional Connectives*

$d_2 = 1.62$ 2. *Verb Use*

$d_3 = -1.27$ 3. *Reduced 1st POV*



Insight Model: w/o & with Gesture

Evidence for the “gist” of the proof



Expertise: $d = 1.07$
 $d_G = 1.00$

Interaction

Marginal: *Dyn. Gesture*
only helps experts



Intuition Model: w/o & with Gesture

“Snap judgment” response of the truth



Expertise: $d = 0.62$
 $d_G = 0.57$

Interaction

Experts w ESL benefit
most from gestures

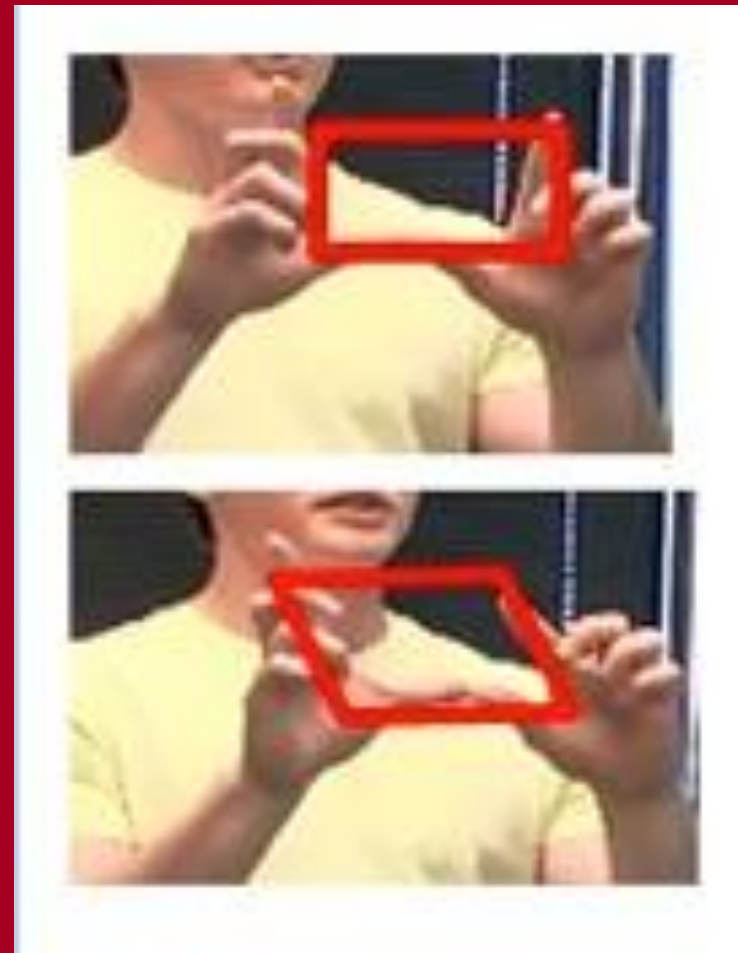


Study 1: Discussion

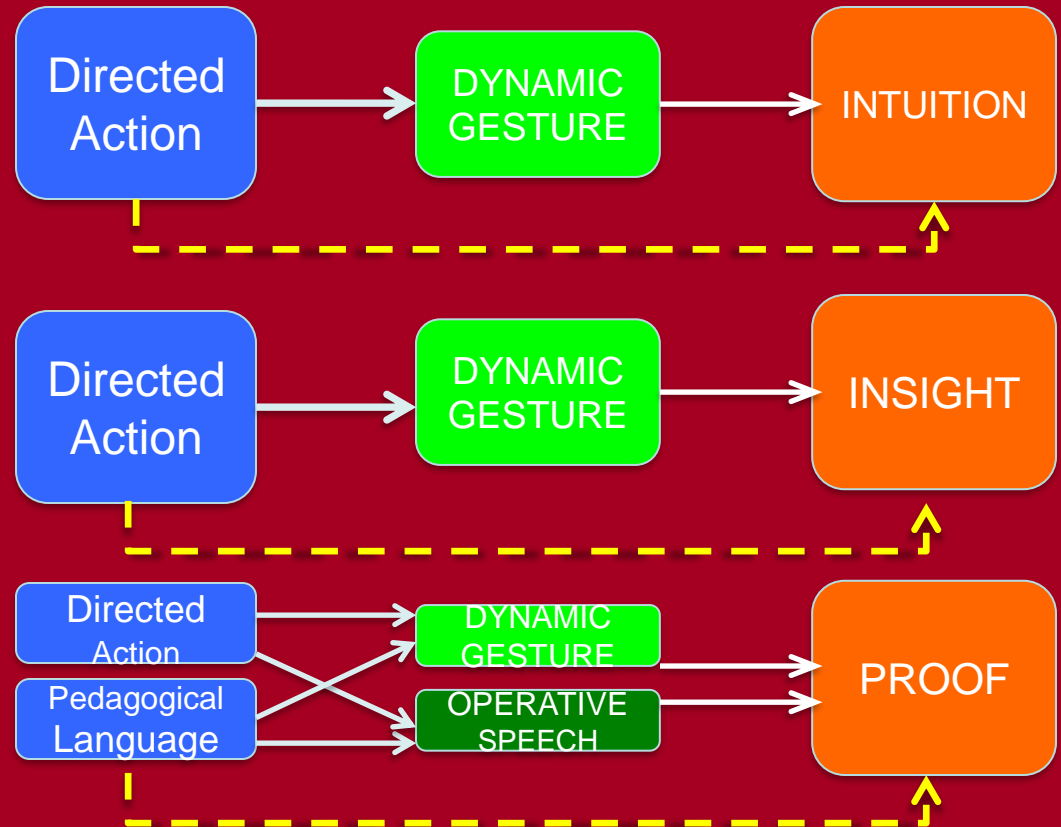
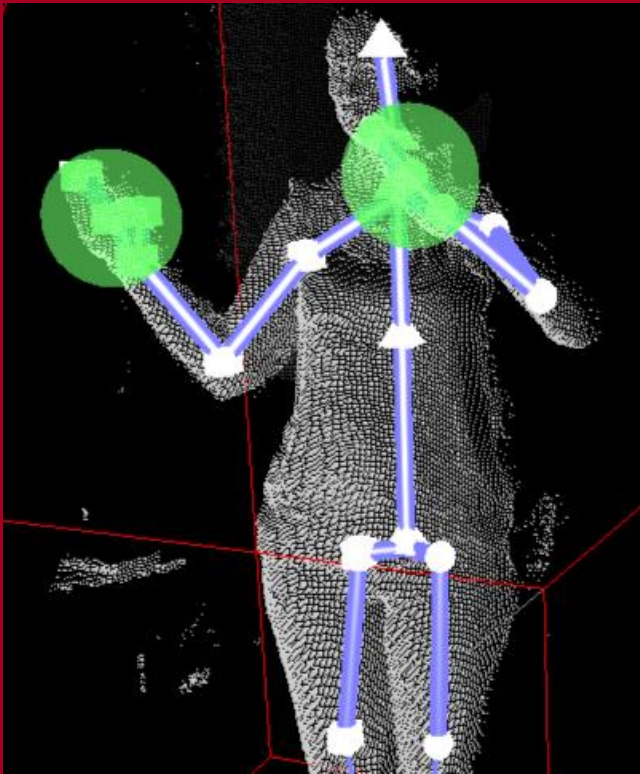
Spatial reasoning &
Dynamic gestures
predict intuition, insight
& informal proofs

Above & beyond
expertise and speech

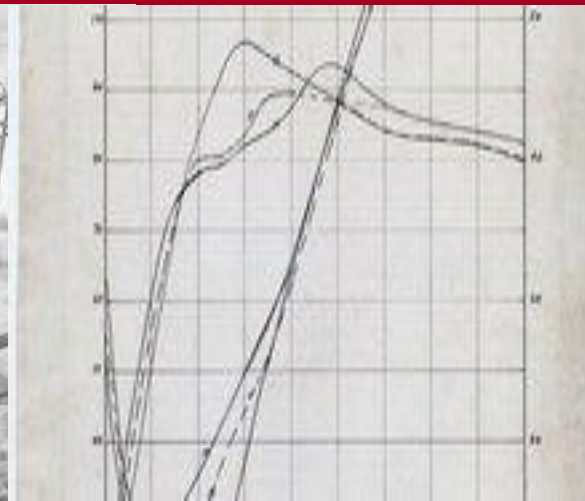
Dynamic gesture
'replaces' spatial reason



Can we Design Interventions?



Meta Design: Near Decomposability





Pilot Study: Video Game



- N = 35 6th-11th grade
- Plays 6 conjectures
 - directed actions
(random) relevant or irrelevant
- After, pedagogical hints linking relevant directed actions to conjectures
- Second attempt





test text test text test text

Back Next

Oddly, the light leads into a doorway through the back of the tent. You move through the opening and see another imposing man sitting near a fire pit. Noticing your presence, he beckons you to join him by the fire. As you sit next to him, the man says, "Follow my moves, young one. Only then can you find spiritual enlightenment."

test text test text test text

test text test text test text

test text test text test text

Consider the statement below.

Given that you know the measure of all three angles of a triangle, there is only one unique triangle that can be formed with these three angle measurements.

When you are ready to proceed select "next."

Next

test text test text test text

Explain why the statement is **always true** or is **false**

Speak your answers out loud as we voice record your responses:

Given that you know the measure of all three angles of a triangle, there is only one unique triangle that can be formed with these three angle measurements.

Recording

Next

test text test text test text

Given that you know the measure of all three angles of a triangle, there is only one unique triangle that can be formed with these three angle measurements.

- A) TRUE: The measurement of all of the angles in a triangle adds up to 180 degrees.
- B) FALSE: You can form different triangles by rotating the triangle.
- C) FALSE: Even if you know the three angles of a triangle, you can make smaller or larger triangles that have the same angles. These would be similar triangles.
- D) TRUE: Once you know the measurement of three angles of a triangle, the sides can only fit together in one unique way.

test text test text test text

Back Next

The chieftain lowers his hands and nods his approval. "You have done well. It is not long now until your body and mind soar from my village." As you gaze into the fire, thinking of home you suddenly see the embers morph. From amongst the flames, the seventh symbol bursts into smoke, leaving only a trail of light behind.



Stage 1:
Observe dynamic
gestures of
competent
problem-solver



“The bottom stays the same, but the sides extend to keep the same height as the rectangle...”

((Dynamic gesture of two vertical sides folding down))



Possibly redes.

Stage 1:
Observe dynamic
gestures of
competent
problem-solver



Stage 2:
Implement “bigger”
version of gestures
as directed actions
in game



“The bottom stays the same, but the sides extend to keep the same height as the rectangle...”

((Dynamic gesture of two vertical sides folding down))

Actions are within game storyline



Possibly redesign

Stage 1:
Observe dynamic
gestures of
competent
problem-solver

Stage 2:
Implement “bigger”
version of gestures
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in game

Stage 3:
Novices perform and
reflect upon directed
actions in context of
problem



“The bottom stays the same, but the sides extend to keep the same height as the rectangle...”

((Dynamic gesture of two vertical sides folding down))

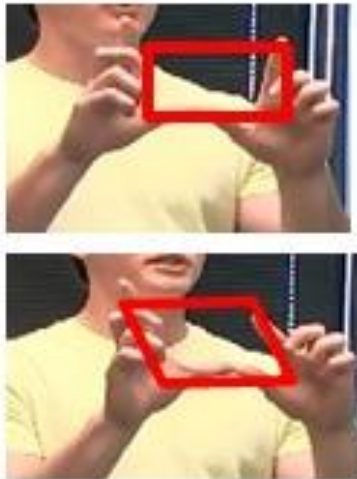


*Actions are within
game storyline*

“This would be the rectangle” (top) “and this would be the parallelogram” (bottom).

Possibly redesign

Stage 1:
Observe dynamic
gestures of
competent
problem-solver



“The bottom stays the same, but the sides extend to keep the same height as the rectangle...”
((Dynamic gesture of two vertical sides folding down))

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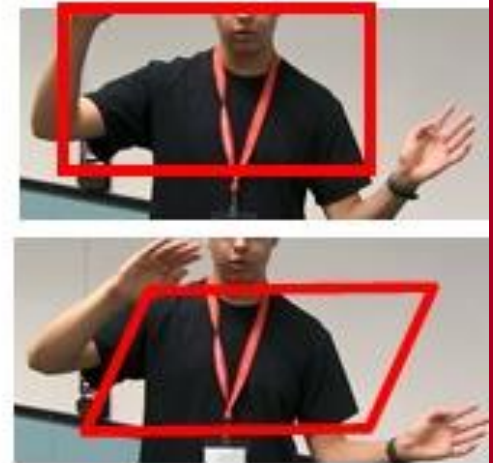
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Stage 3:
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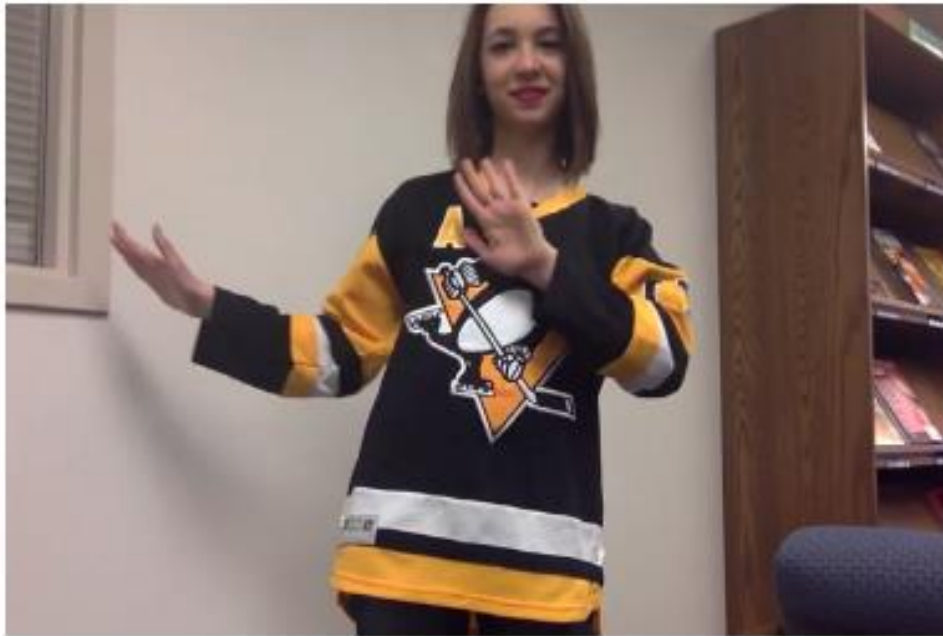


“This would be the rectangle” (top) “and this would be the parallelogram” (bottom).

Stage 4:
Novices are able to
transform directed
actions into co-speech
dynamic gestures.

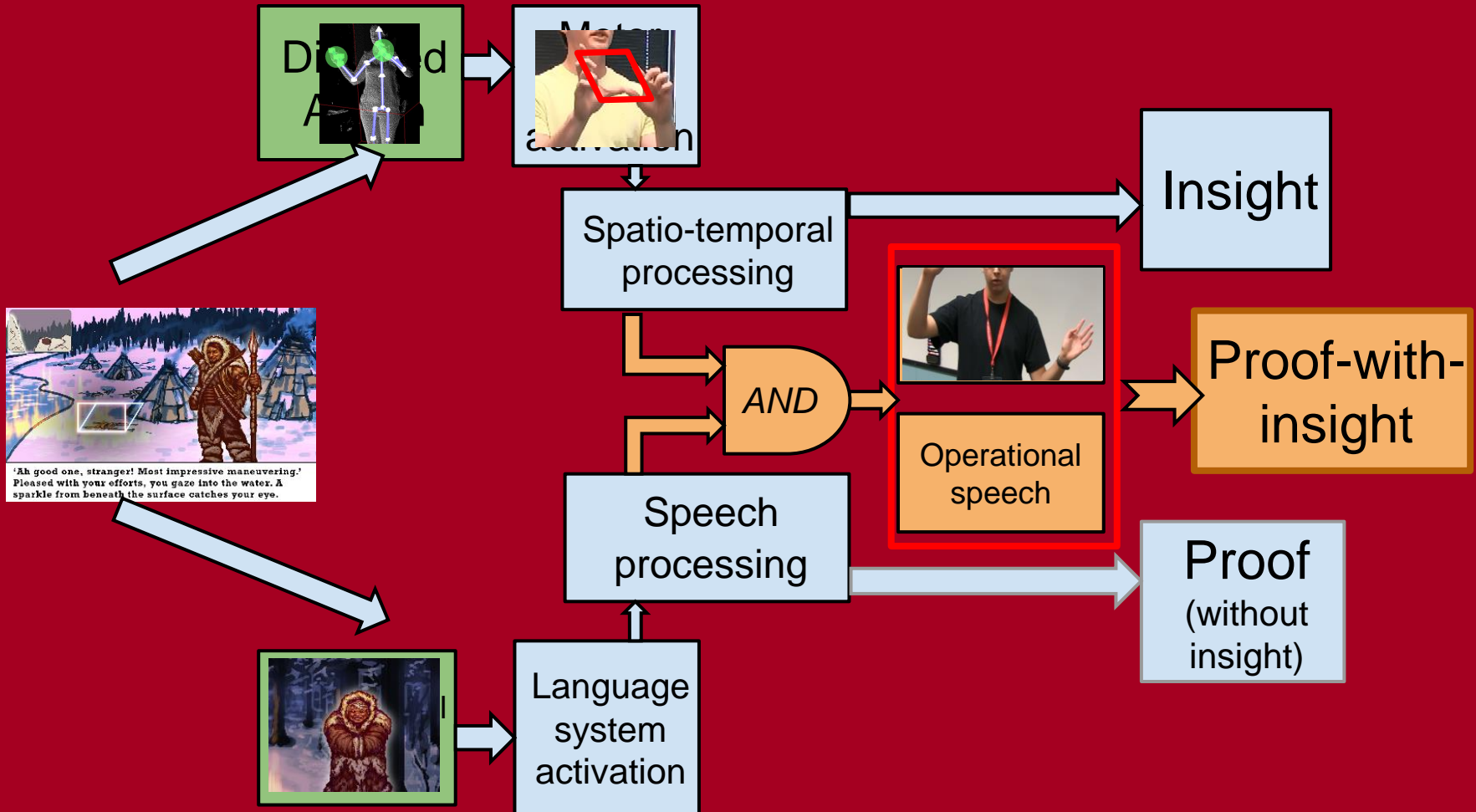


“If it’s the same length and height, the parallelogram is just displaying it differently. Instead of it all being perpendicular and parallel, all the angles are a little bit skewed but they’ll still make up... all the same area.”
((Dynamic gesture of two vertical sides folding down))





Logic Model





Implications

- Theory based embodied design
 - Grounding role of action and language
 - Dynamic gestures mediate intuition & proof
- People really do more math than they may think
 - Channel intuitions via perceptual motor processes

Opportunities thru new video game experiences



Acknowledgements



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